10/1/18

GAME THEORY - WEEK 1

FOCUS: ABSTRACT REASONING
DISCIPLINING REASONING W/ MATHEMATICS
THE INTERNAL LOGIC OF MODELS
DISCIPLINES OUR REASONING
"GAME THEORY DISCIPLINES THE CONCEPTS WE USE."

HERNAN
TERRY
VITOR
ADAM WEI
KEVIN (ATLANTA AREA)
PATRICK BIANCA
YIAN GEOFF
KENNEDY TOM

ANALOGY W/ A LANGUAGE COURSE -> LOW GOAL IS TO KNOW VOCAB, ETC. BUT NOT TO WRITE MODELS
"IF YOU CAN'T WRITE IT, YOU DON'T KNOW IT."

* REFLECTION AT THE END OF PROBLEM SETS
GROWTH MINDSET VS. GRADES MINDSET
-> PEOPLE WORKED WITH, WEBSITES CONSULTED
TABLESGENERATOR.COM - MATRICES + TABLES
NO BLANKS ON HW -> TRY EVERYTHING TO GET A
QUIZZES ON THE MONDAY AFTER PROBLEM SETS

KURAN CITES DAVID HACKETT FISHER ON HISTORIANS' FALLACIES
"POLITICAL REVOLUTIONS WILL CONTINUE TO SURPRISE US" B/C OF PREFERENCE FALSIFICATION
SCHELLING: MODELS ARE 1) A PRECISE + ECONOMICAL STATEMENT OF A SET OF RELATIONSHIPS OR AN ACTUAL BIOLOGICAL, MECHANICAL, SOCIAL SYSTEM

WHY MODELS?

GAME THEORY - "HOW INSTRUMENTAL RATIONALITY MANIFESTS IN SITUATIONS OF STRATEGIC INTERDEPENDENCE"

IN A STRATEGIC CONTEXT, WITH MULTIPLE PEOPLE PURSUING GOAL ORIENTED BEHAVIOR, WE CAN'T JUST WORRY ABOUT THE PROBABILITIES (PARAMETRIC REASONING), WE HAVE TO USE MODELS TO EMPLOY STRATEGIC REASONING

MODEL $\neq$ THEORY

↳ THIS IS A CLASS ABOUT MODELS (OF RATIONALITY)

MODELS ARE BOTH CONCRETE (SPECIFIC SITUATIONS) AND ABSTRACT (B/C THEY ARE MATHEMATICALLY BROAD + CAN HAVE MULTIPLE APPLICATIONS).

KURAN - "LATENT REVOLUTIONARY BANDWAGON"

{0, 20, 20, 30, 40, 50, 60, 70, 80, 100}

{0, 10, 20, 30, 40, 50, 60, 70, 80, 100}

REVOLUTION! NO REVOLUTION

DEVELOPMENTS IN POLISCI OFTEN HAPPEN BY ANALOGIZING FROM ONE DISCIPLINE (LEGISLATIVE BARGAINING) TO ANOTHER CONTEXT (CRISIS BARGAINING).

WED - BEGIN w/ SECTION 2 OF NOTES

10/2/18

GAME THEORY: TADELIS CH 1

DECISION PROBLEMS: 1) ACTIONS
2) OUTCOMES
3) PREFERENCES

PREFERENCE RELATIONS: $x \succsim y$: "x IS AT LEAST AS GOOD AS Y"
↳ CAN BE EITHER:
STRICT PREFERENCE RELATION: $x \succ y$: "x IS STRICTLY BETTER"
INDIFFERENCE RELATION: $x \sim y$: "x AND y ARE EQUALLY GOOD"

COMPLETENESS AXIOM: ANY TWO OUTCOMES x, y CAN BE RANKED BY THE PREFERENCE RELATION SO THAT EITHER $x \succsim y$ OR $y \succsim x$.
↳ DOES NOT LET A PLAYER BE INDECISIVE
↳ ELIMINATES BURIDAN'S ASS

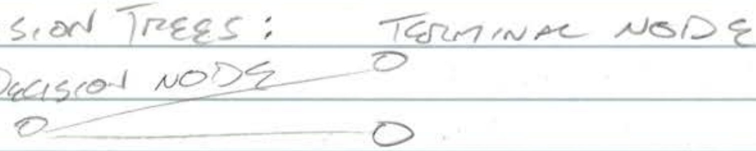
TRANSITIVITY AXIOM: FOR ANY 3 OUTCOMES x, y, z, IF $x \succsim y$ AND $y \succsim z$, THEN $x \succsim z$

RATIONAL PREFERENCE RELATION: COMPLETE + TRANSITIVE
* NOTE CONDORCET PARADOX - RATIONAL PLAYERS IN A GROUP CAN FAIL TO REACH A DECISION
"IMPOSING INDIVIDUAL RATIONALITY DOES NOT IMPLY GROUP RATIONALITY

PROFIT/PAYOFF FUNCTION (RATIONAL PLAYERS ONLY)
EVERY ACTION $a \in A$ YIELDS A PROFIT $\pi(a)$
PAYOFF VALUES ARE ORDINAL -> NO ABSOLUTE VALUE
REPRESENTED BY $u(\cdot)$ OR $v(\cdot)$
PLAYERS W/ RATIONAL PREFS WILL MAXIMIZE THIS FUNCTION

PROPOSITION 1.1 => IF THE SET OF OUTCOMES X IS FINITE, THEN ANY RATIONAL PREFERENCE RELATION OVER X CAN BE REPRESENTED BY A PAYOFF FUNCTION.

DECISION TREES:



RATIONAL CHOICE PARADIGM
↳ ASSUMES FULL KNOWLEDGE OF POSSIBLE ACTIONS, OUTCOMES, AND PREFERENCES

PAYOFFS OVER ACTIONS: IF ACTION a LEADS TO OUTCOME $x(a)$, THEN THE PAYOFF FROM A IS GIVEN BY $v(a) = u(x(a))$, SO $v(a)$ = PAYOFF OF a

CONTINUOUS ACTION SPACE:
1) DEFINE PAYOFF FUNCTION
2) MAXIMIZE - TAKE 1ST DERIVATIVE + SET = 0

10/3/18

ETHICS + SOCIETY 13

TOURSCI OFFICE CAN GIVE A LIST OF NAMES FOR SECTIONS

MORALITY IS ABOUT ACCOUNTABILITY - WE OUGHT TO BE ABLE TO HOLD OTHERS TO ACCOUNT.

SLIDE: PARALLELS BTW. MUSIC THEORY + MORAL THEORY
↳ WE HAVE USEFUL MORAL INTUITIONS, BUT (PACE BLOOM) THEY NEED TO BE DISCIPLINED BY THEORY

REVIEW IS-OUGHT DISTINCTION - ANY PROBLEMS?
↳ IS SCIENCE REALLY ONLY "IS"?

SEE HARRY SHEARER(?) ON IS-OUGHT

FOLK PSYCHOLOGY - USE RUSSELL ON THE GENERALIZABILITY OF IDEALS → SOMETHING WE WANT OTHERS TO WANT

JUSTICE - REVIEW MACKIE'S TAXONOMY
↳ DISTRIBUTIVE, COMMUTATIVE, ETC.

REVIEW SYLLABUS - HOW ARE THEY BEING EVALUATED?

DEONTIC LOGIC? SUPEREROGATORY? ("KINDS OF MORAL ACTIONS")

→ PREFERENCE VS. OBLIGATION → IDEALS (AND NEG. IDEALS) ARE UNIVERSALIZABLE

HELP THEM SEE THE FOREST - PLAY THE VIDEOS HE SKIPPED
WHAT IS A TRITON? - CLEAN SHOES!

10/3/18

GAME THEORY WEEK 1 READING

1) INSTRUMENTAL RATIONALITY - MEANS-END
STRATEGIC INTERDEPENDENCE - DETERMINING ONE INDIVIDUAL'S OPTIMAL STRATEGY REQUIRES TAKING INTO ACCOUNT OTHERS' STRATEGIES.

DECISION THEORY BECOMES GAME THEORY ONCE YOU BRING IN STRATEGIC INTERDEPENDENCE.

2) A THEORY OF INSTRUMENTAL RATIONALITY MUST:
1) CAPTURE AN INTUITIVE IDEA OF CONSISTENT, GOAL-DIRECTED BEHAVIOR
2) SIMPLE - FOCUS ON ESSENTIAL ASPECTS
3) USEFUL - YIELDS NON-OBVIOUS INSIGHTS
4) RICH - INDICATES NEW APPROACHES
5) FLEXIBLE - WIDELY APPLICABLE

IT NEED NOT BE REALISTIC - PPL AREN'T ALWAYS RATIONAL

3) GOAL-DIRECTEDNESS CAN BE MODELED AS PREFERENCES OVER A SET OF POSSIBLE OUTCOMES
1) OUTCOMES: $X = \{x_1, \ldots x_N\}$ N OUTCOMES, INDEXED BY i (THE SET OF OUTCOMES IS FINITE)
2) PREFERENCES: REPRESENTED BY A RANKING OF OUTCOMES: $R = \{(x,y) : x, y \in X\}$ (ORDER MATTERS, NEED NOT INCLUDE ALL OUTCOMES)

INSTEAD OF $(x,y) \in R$, WE CAN WRITE
xRy OR $x \succsim y$ - "x IS WEAKLY-PREFERRED TO y"
$x \succ y$: "x IS STRICTLY PREFERRED TO y" (xPy)
$x \sim y$: "x IS INDIFFERENT TO y" (xIy)

ACTIONS: $A = \{a, b, c, \ldots\}$ OR $A = \{a_1, a_2, \ldots a_k\}$
(FINITE, FEASIBLE, CAN INCLUDE $(a+b)$ OR $(a \vee c)$)

4) CHOICE SET: $C(\succsim_i X) \subseteq X$ (PERSON i'S CHOICES)
↳ SUBSET OF OUTCOMES i PREFERS AS MUCH OR MORE THAN OTHER OUTCOMES, DEFINED RELATIVE TO A PREF RELATION ($\succsim_i$) AND AN OUTCOME SET (X)

* (REVIEW EX. 4.1.1-2)

$X \setminus \{w\}$ INCOMPLETE PREFS: THE CHOICE SET IS INCOMPLETE WHENEVER PREFERENCES ARE 1) CYCLIC OR 2) INCOMPLETE

COMPLETENESS: A PREFERENCE RELATION IS COMPLETE IFF FOR ALL $x, y \in X$, $x \succsim y$ OR $y \succsim x$
↳ $\forall x, y \in X (x \succsim y \vee y \succsim x)$

TRANSITIVITY: A PREFERENCE RELATION IS TRANSITIVE IFF FOR ALL $x, y, z \in X$, $x \succsim y \wedge y \succsim z$ MEANS $x \succsim z$

(* →) ↳ $\forall x, y, z \in X[(x \succsim y) \wedge (y \succsim z) \Rightarrow x \succsim z]$

A RELATION THAT'S COMPLETE + TRANSITIVE IS CALLED AN ORDERING, A WEAK ORDER, OR A TOTAL PREORDER

* QUASI-TRANSITIVITY: $\forall x, y, z \in X[(x \succ y \wedge y \succ z) \Rightarrow x \succ z]$

DEFINITIONS: $x \succsim y \Leftrightarrow \neg(y \succ x)$
$x \sim y \Leftrightarrow (\neg(y \succ x) \wedge \neg(x \succ y))$

ASYMMETRIC RELATIONS:
(*) (≡) $\forall x, y \in X \, \neg(x \succ y \wedge y \succ x) \equiv \forall x, y \in X[x \succ y \rightarrow \neg(y \succ x)]$

NEGATIVE TRANSITIVITY:
$\forall x, y \in X[x \succ y \rightarrow \forall z \in X(x \succ z \vee z \succ y)]$

5) CONTINUOUS (INFINITELY DIVISIBLE) OUTCOME SET:
↳ COMPACT IF CLOSED + BOUNDED

IF X IS INFINITE + NON-EMPTY, THEN IF X IS COMPACT AND $\succsim$ IS COMPLETE, TRANSITIVE + L.C., THEN $C(\succsim, X) \neq \emptyset$

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GAME THEORY - WEEK 1B CLASS NOTES

READ MEYERSON ARTICLE ON SCHEDULING

MEANS - END RATIONALITY $\rightarrow$ CONSISTENT, GOAL-DIRECTED BEHAVIOR

ACTION / OPTION \ OUTCOME: $X = \{x_1, x_2 \ldots x_n\}$

OUTCOMES PLUS PREFERENCES: ENDS

ACTIONS: MEANS $\quad A = \{a_1, \ldots, a_k\}$

$R \rightarrow$ BINARY RELATIONS (ORDERED PAIRS)

$(x,y) \neq (y,x)$

PRODUCT SET $X \cdot X = $ (x,x) (y,z) (x,y) (z,z) (x,z) (z,x) (y,x) (z,y) (y,y)

$X = \{x, y, z\}$ $\quad X = \{x, y, z\}$

$xRy = x \succsim y$ (\succsim IN LATEX)

(OUTCOMES: NODES
ORDERING: EDGES

OUTCOMES: $X = \{x, y, z\}$

PREFERENCES: $x \succsim y$, xRy

$x \succ y \equiv x \succsim y \land \neg(y \succsim x)$

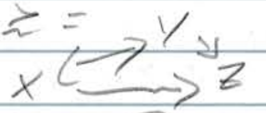
$x \sim y \equiv x \succsim y \land y \succsim x$

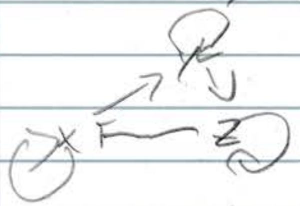
AGENTS DO NOT HAVE PREFERENCES OVER ACTIONS, ONLY OVER OUTCOMES.

CHOICE SET : $C(\succsim, X)$

↳ ALWAYS DEFINED BY 1) CHOICE SET AND 2) OUTCOMES

$C(\succsim, X) := \{x \in X : \forall y \in X\, [x \succsim y]\}$

$\succsim =$  : $C = \{x, y\}$

 : $C = \{\emptyset\}$ (EMPTY SET DOESN'T MEAN INDIFFERENCE)

NON-EMPTY CHOICE SETS CAN ARISE FROM
1) INCOMPLETENESS , AND 2) CYCLING

COMPLETENESS: $\forall x, y \in X\ (x \succsim y \lor y \succsim x)$
TRANSITIVITY : $\forall x, y, z \in X\ [(x \succsim y \land y \succsim z) \rightarrow x \succsim z)$
EDGE BTW. EVERY NODE

IF X IS FINITE + NON-EMPTY AND $\succsim$ IS COMPLETE AND TRANSITIVE, THEN THE CHOICE SET IS NOT EMPTY
PROOF: BY INDUCTION
ASSUME X IS FINITE + NON-EMPTY
BASE CASE : $X = \{x\}$ $\quad C(\succsim X) = \{x\}$
INDUCTION : $X' = \{x_1, \ldots x_m\}$ $C(\succsim X) \neq \{\emptyset\}$
$X = X' \cup \{a\}$
$= \{x_1, \ldots x_m, a\}$
TWO CASES : $a \succsim x$ OR $x \succsim a$
BY TRANSITIVITY
$C(\succsim, X) \neq \emptyset$ $\quad$ ASK *
(AT LEAST x)

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GAME THEORY 2a - NOTES

FUTURE CLASSES IN THE DEAN'S CONFERENCE ROOM
↳ CIRCULAR COURTYARD, UP THE STAIRS
TO THE LEFT

REVIEW

BASE : $X = \{x\}$
STEP 1 : $X = \{x\} \cup \{a\} = \{x, a\}$ } $a \succsim x$, $x \succ a$

CASE 1: $a \succsim x$ } IN BOTH CASES, CHOICE
CASE 2: $x \succ a$ } SET IS FINITE + NON-EMPTY

COMPLETENESS + TRANSITIVITY ARE SUFFICIENT CONDITIONS FOR A NON-EMPTY CHOICE SET.

TRANSITIVITY

P	P -> Q	Q
T	T	T
T	F	F
F	T	T
F	T	F

QUASI - TRANSITIVITY
DEFINED ON BINARY RELATION

x y
180 ~ 179
179 ~ 178 z
178 ~ 177

ACYCLICITY - NO CYCLES, DEFINED ON EVERY SUBSET OF OUTCOME SET.

W -> X -> Y -> Z

NOT ALLOWED

W -> X -> Y -> Z   W -> X -> Y -> Z   ALLOWED

ASK DW TO SEND PROOF OF 4.2

TRANSITIVITY $\xrightarrow{\text{IMPLIES}}$ QUASI-TRANS $\xrightarrow{\text{IMPLIES}}$ ACYCLICITY

4.2 ACYCLICITY IS BOTH NECESSARY + SUFFICIENT FOR CONSTRUCTING A NON-EMPTY CHOICE SET.

PROOF: SUPPOSE X IS FINITE + NON-EMPTY
SUPPOSE $\succsim$ ON X IS COMPLETE

SHOW $C(\succsim, X) = \{\emptyset\}$ IFF $\succsim$ IS ACYCLIC

$P \Longleftrightarrow Q$

① IF $C \neq \{\emptyset\}$, THEN $\succsim$ ACYCLIC

$P \to Q$ — ASSUME ANTECEDENT!

REDUCTIO AD ABSURDUM

ASSUME $C \neq \{\emptyset\}$

ASSUME $\succsim$ NOT ACYCLIC

$X = \{w, x, y, z\}$ $C = \{w\}$

SAY $w \to x \to y \to z$

IF $C = \{w\}$, THEN THERE CAN'T BE A CYCLE

~~TWO~~ THREE MOST PROMINENT STRATEGIES FOR PROVING $P \to Q$

ASSUME ANTECEDENT (SEE VELLEMAN)

PROVE CONTRAPOSITIVE

AND ASSUME NEGATION OF CONSEQUENT

② IF $\succsim$ ACYCLIC, THEN $C \neq \{\emptyset\}$

$w \to x$ y

$y \succsim w$ OR $\neg(y \succsim w)$ $w \succ y$

EITHER $w \to x \to y$

OR $w \to x \leftarrow y$

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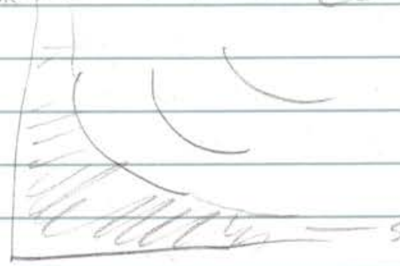
5) CONTINUOUS CASE:

COMPACT: BOUNDED + CLOSED (ENDS ARE IN SET)

* INDIFFERENCE CURVES



6) $u(x) \geq u(y)$ IFF $x \succsim y$

RELATION ON NUMBERS   RELATION ON OUTCOMES

$$u(\emptyset) = \begin{cases} 7 & \emptyset = x \\ 3 & \emptyset = y \end{cases}$$

$$\left.\begin{array}{ll} u(x) > u(z) & x > z \\ u(x) > u(y) & x > y \\ u(y) = u(z) & y \sim z \end{array}\right\} \text{REPRESENTATION}$$

WHAT MUST BE TRUE OF A PREFERENCE RELATION FOR IT TO BE REPRESENTED AS A UTILITY FUNCTION?

$\exists\, u: X \to \mathbb{R}$ THAT REPRESENTS $\succsim$ IFF THAT PREFERENCE RELATION IS COMPLETE + TRANSITIVE.

(INTUITION: THE $\geq$ AND $\leq$ RELATIONS ARE ALSO COMPLETE + TRANSITIVE)

PROOF: TAKE SOME FINITE X

FOR x IN X_i, $\underline{v(x) = k - i}$

(REPRESENTATION THEOREM)

ORDINAL FUNCTIONS, IF TRANSFORMED, CONVEY THE SAME INFORMATION.

SATIABLE PREFERENCES CAN BE MODELED BY QUADRATIC $(-(t-t^*)^2$ OR $-|t-t^*|$

(DIFFERENTIABLE) (NOT DIFFERENTIABLE)

(SPATIAL PREFERENCES)

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GAME THEORY 2B NOTES

CARDINAL UTILITY FUNCTIONS

	COLD (.2)	MILD (.5)	HOT (.3)
BEACH	0	1	2
MOVIE	2	0	1
HIKE	1	2	0

OUTCOMES
(BEACH, COLD)
(MOVIE, HOT)
ETC.

TAKE THE PROBABILITY - WEIGHTED SUM OF VALUES

BEACH: $0(.2) + 1(.5) + 2(.3) = 1.1$
MOVIE: $2(.2) + 0(.5) + 1(.3) = .7$
HIKE: $1(.2) + 2(.5) + 0(.3) = 1.2$

THIS IS SUSCEPTIBLE TO A PROBLEM - TRANSFORMATION OF THE DATA WILL YIELD A DIFFERENT RESULT

SAME PREFERENCE RELATIONS, DIFFERENT NUMBERS

	COLD (.2)	MILD (.5)	HOT (.3)
BEACH	0	3	4
MOVIE	4	0	3
HIKE	3	4	0

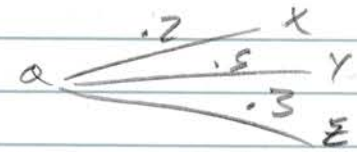
BEACH = 2.7
MOVIE = 1.7
HIKE = 2.6

CARDINAL UTILITY FUNCTION: $u(x) - u(y) > u(x) - u(z)$
LOTTERIES: PROBABILITY DISTRIBUTIONS OVER OUTCOMES
BERNOULLI UTILITY FUNCTION: $u: X \to \mathbb{R}$; $\succeq$ on X
SET OF LOTTERIES: P ; INDIV. LOTTERIES: $\{p, q, r\}$
OR $\{p_1, p_2, p_3\}$

DEGENERATE LOTTERY : ONE OUTCOME HAS P=1

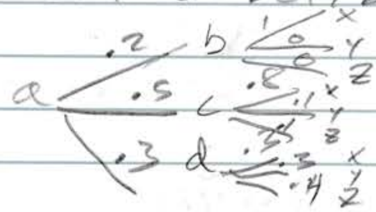
LOTTERIES ARE REPRESENTED AS VECTORS:
↳ $(0.2, 0.5, 0.3 ; x, y, z)$
OR AS TREES:

FIG 4.0



CAN BE ADAPTED TO LOTTERIES OVER LOTTERIES (COMPLEX)

COMPLEX LOTTERY : REDUCED LOTTERY



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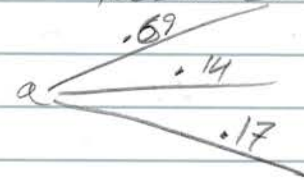
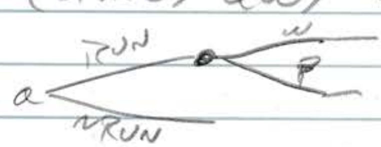


CONSISTENT, GOAL-DIRECTED AGENTS CHOOSE THEIR MOST-PREFERRED LOTTERY.
(EXPECTED UTILITY MAXIMIZATION)

$p \succsim q$ IF $p(x)u(x) + p(y)u(y) + p(z)u(z) \geq$ SAME FOR q
(LOTTERIES) (NUMBERS) $U(p) \geq U(q)$

RUNNING FOR OFFICE
u(RUNNING & WINNING) $= 10 - 5 = 5$
u(RUNNING & LOSING) $= 0 - 5 = -5$
u(STATUS QUO) $= 0$



$0.4(x-5) + 0.6(-5)$

HOW MUCH DOES RUNNING FOR OFFICE HAVE TO RETURN?

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PREFERENCES OVER LOTTERIES:
$\succsim$ ON $\mathcal{P}$ (PREFERENCES OVER LOTTERIES)

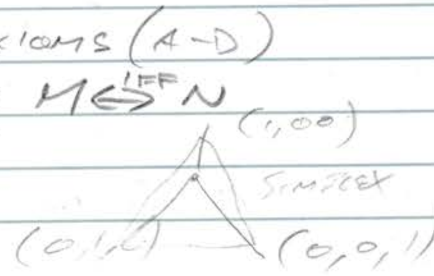
PROOF OF PROPOSITION 9.1 VNM THEOREM
M ↓

M 9.1 → [A PREFERENCE RELATION ON LOTTERIES SATISFIES CONDITIONS (A, B, C, D)] ONLY IF (IFF)
N [THERE EXISTS A CARDINAL UTILITY RELATION $(u: X \to \mathbb{R})$ SUCH THAT FOR ALL LOTTERIES IN THE SET OF LOTTERIES $(\forall p, q \in \mathcal{P})$
$p \succsim q$ IFF $\sum p(x) u(x) \geq \sum q(x) u(x)$]

WHAT ARE AXIOMS A-D?
A: RATIONAL (COMPLETE, TRANSITIVE)
B: REDUCTION OF COMPOUND LOTTERIES
↳ PLAYERS CARE ABOUT OUTCOMES
C: CONTINUITY – $p \succ q \succ r$
$[\alpha p + (1-\alpha) r] \succsim q$; $[\beta p + (1-\beta) r] \precsim q$
THE SET OF ALPHAS SUCH THAT p IS PREFERRED TO q IS CLOSED
CONTINUITY RULES OUT LEXICOGRAPHICAL PREFERENCES
↳ EXTREME PREFERENCES ⇒ NOT WILLING TO GAMBLE
D: INDEPENDENCE: PREFERENCES OVER TWO LOTTERIES SHOULD REMAIN FIXED, EVEN IF WE MIX THE SAME THIRD LOTTERY WITH BOTH OF THE FIRST TWO.

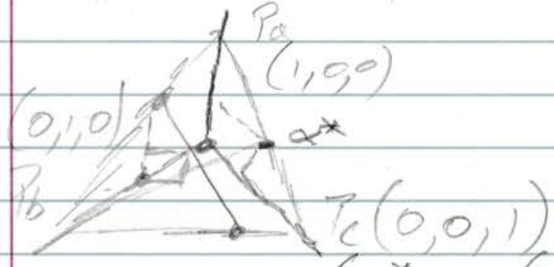
—

VON NEUMANN – MORGANSTERN AXIOMS (A–D)
TO PROVE (9.1) → STRUCTURE: $M \overset{IFF}{\Leftrightarrow} N$
ASSUME THE AXIOMS: $A \succ B \succ C$



PROB SIMPLEX IS THE FACE FACING US
→ PROBS ARE SUM TO 1

GAME THEORY 3A NOTES 10/15/12



$(\alpha^*, 0, (1-\alpha^*))$

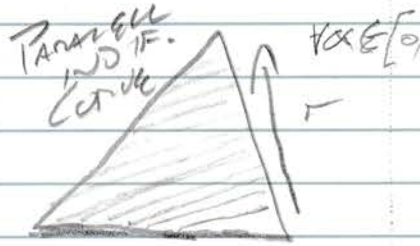
$$(0,1,0) \sim [\alpha^*(1,0,0) + (1-\alpha^*)(0,0,1)]$$

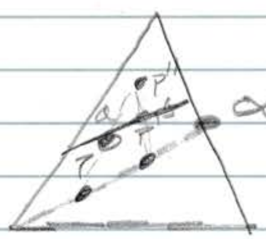
$$(0,1,0) \sim [\beta(0,1,0) + (1-\beta)(0,1,0)]$$

$$(0,1,0) \sim [\beta(0,1,0) + (1-\beta)[\alpha^*(1,0,0) + (1-\alpha^*)(0,0,1)]]$$

$$\beta(0,1,0) + (1-\beta)\alpha^*(1,0,0) + (1-\beta)(1-\alpha^*)(0,0,1)$$

2 LOTTERIES





α^* $a[\frac{1}{2}p, \frac{1}{2}p'']$ (VIA INDEPENDENCE AXIOM)
$b[\frac{1}{2}p', \frac{1}{2}p'']$

9.4.1 FOR ANY α, THERE'S AN INDIFFERENCE CURVE

$U(p) = \alpha_p$; $U(p) \geq U(q)$ IFF $p \succsim q$

$U(p) = U(q)$ IFF $p \sim q$

$U(p) \leq U(q)$ IFF $p \precsim q$

$$U(p) = \sum_{x \in X} p(x)\, u(x)$$

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GAME THEORY 3A NOTES CONT'D

WE CAN NOW USE A CARDINAL UTILITY FUNCTION TO REPRESENT PREFERENCES OVER LOTTERIES (AND THUS OVER OUTCOMES).

9.5 IF $u: X \to \mathbb{R}$ THAT REPRESENTS $\succsim$ ON X, THEN $v(u(x)) = a(u(x)) + b$ ALSO REPRESENTS $\succsim$ ON X. (a MUST BE POSITIVE, b ANYTHING)

↳ THE ONLY KIND OF TRANSFORMATION THAT PRESERVES CARDINAL UTILITY FUNCTIONS ARE LINEAR (AFFINE) TRANSFORMATIONS - THEY SHIFT THE SLOPE AND/OR Y INTERCEPT.

$$U'(P) = a \cdot U(P) + b$$

PREFERENCES ARE STILL PRIOR TO UTILITIES. I DO NOT PREFER a OVER b BECAUSE IT HAS A HIGHER UTILITY, BUT RATHER a HAS A HIGHER UTILITY <u>BECAUSE</u> I PREFER IT.

WHY ON EARTH WOULD YOU ASSIGN A QUIZ QUESTION BASED ON HW Q'S MOST PPL GOT WRONG <u>BEFORE</u> REVIEWING?

STRATEGIC INTERDEPENDENCE
↳ INTRODUCING OTHER CHOOSING AGENTS
HOW DO WE MODEL THIS?

COOPERATIVE GAME: WHERE A "PRE-PLAY" STAGE PROVIDES AN OPPORTUNITY TO MAKE A (BINDING) AGREEMENTS ABOUT COOPERATION

NON-COOPERATIVE GAME: NO PRE-PLAY COLLUSION POSSIBLE, BUT COOPERATION IS STILL POSSIBLE, JUST NOT ENFORCEABLE
↳ OUR FOCUS WILL BE NON-COOPERATIVE GAMES (GAMES OF STRATEGY)

ELEMENTS: PLAYERS, ACTIONS, OUTCOMES, INFORMATION, TIMING

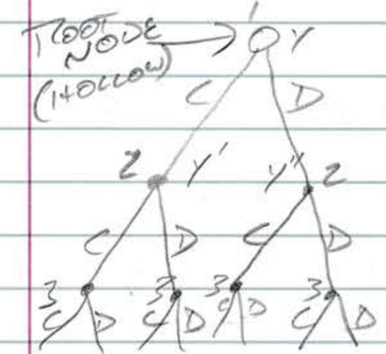
PRISONER'S DILEMMA: STANDARD EXPOSITION
$I = \{1, 2, 3 \ldots n\}$ (PLAYERS)
u_1 = UTILITY OF PLAYER 1; u_i = UTILITY OF P_i

READ TO 3.5

10/17/18

GAME THEORY 3B - NOTES

SEQUENTIAL



PRISONER'S DILEMMA

NODES: $Y = \{Y, Y', Y'' \ldots\}$

Y' AND Y'' HAVE NO RELATION

BUT $Y > Y''$

↳ COMPLETE, TRANSITIVE $(Y > Y' > Y'')$

YOU CAN ONLY REACH SUBSIDIARY NODES VIA ONE ROUTE (UNIQUE PREDECESSOR)

ONE PLAYER AT EACH NODE

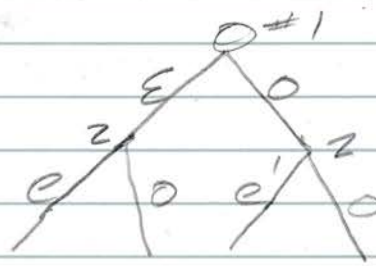
↳ FUNCTION: 1 TO 1

POWER SET: SET OF ALL SUBSETS

↳ SET: $\{A, B, C\}$ POWER SET: $\{\{A\}, \{B\}, \{C\}, \{A,B\}, \{B,C\}$ ... AND $\emptyset\}$

1) ASSIGN NODES TO PLAYERS

2) ASSIGN ELEMENTS OF THE POWER SET TO NODES



(CANDIDATE ENTRY GAME)

ACTION SPACES:

$P_1 : \{E, O\}$

$P_2 : \{e, o, e', o'\}$

EACH OUTCOME IS UNIQUE: $(E, e) \neq (O, e')$

SIMULTANEOUS MOVE GAMES

COMPLETE VS. INCOMPLETE

PERFECT VS. IMPERFECT

CERTAIN VS. UNCERTAIN

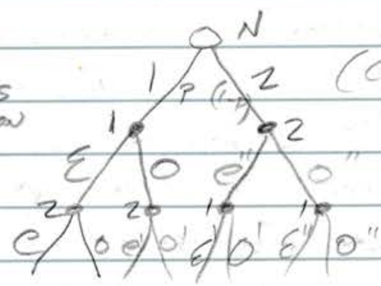
INFORMATION SET: A SET OF NODES REPRESENTING "WHAT A PLAYER KNOWS" → REPRESENTED BY --- DASHED LINES LINKING NODES. WHEN PLAYER 2 MOVES, THEY DON'T KNOW WHICH CONTINGENCY THEY'RE ACTING UNDER.

WHAT WE OBSERVE DEPENDS ON WHAT HAPPENS COUNTERFACTUALLY

IN THE CERTAIN CASE, ALL INFORMATION SETS ARE SINGLETONS. IF THERE'S MORE THAN ONE ELEMENT IS THE INFORMATION SET, THEN THE CHOICE IS UNCERTAIN.

NATURE: RANDOM CHANCE IS MODELED AS A PLAYER IN THE GAME
↳ PROBABILITY DISTRIBUTION

- - - - - DASHED LINES ARE INFORMATION SETS



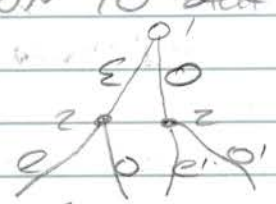
(CHOICE OF FIRST MOVER) (~~PLAYER # IS AN~~ OUTCOME)

UNCERTAINTY: STOCHASTIC EVENT

COMMON KNOWLEDGE: THE PLAYERS HAVE A SHARED UNDERSTANDING OF THE GAME. (THEY ALL KNOW THE FACTS, KNOW THAT EACH OTHER PLAYER KNOWS THE FACTS, AND SO ON.)

PREFERENCES: UTILITY FUNCTIONS (DEFINED OVER OUTCOMES)
BELIEFS: PROBABILITY DISTRIBUTIONS

STRATEGIES: A COMPLETE CONTINGENCY PLAN
↳ ASSIGNS AN ACTION TO EACH NODE



EACH TERMINAL NODE HAS A UNIQUE STRATEGY PROFILE

$S_1 = \{E, O\}$
$S_2 = \{ee', eo', oe', oo'\}$

STRATEGY PROFILE: CARTESIAN PRODUCT OF STRATEGY SETS:

S_1: E, O
S_2: ee', eo', oe', oo'

$= (E, ee')(E, eo')(E, oe')(E, oo')(O, ee')(O, eo')(O, oe')(O, oo')$

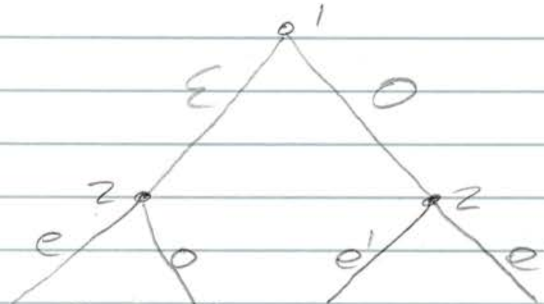
10/22/18

GAME THEORY 4A - NOTES

EXPECTED UTILITY

CRUCIAL CONCEPTS: STRATEGY - COMPLETE, CONTINGENT PLAN FOR ALL PLAYERS

BEST RESPONSE



IF A PLAYER HAS N NODES W/ M STRATEGIES, THEN THEY HAVE N^M STRATEGIES

$S_1 = \{E, O\}$

$S_2 = \{ee', eo', oe', o, o'\}$

$S = (S_1, S_2)$

$s = (s_i, s_{-i})$

$S_1 \times S_2 = S$

S_1	S_2	S	
E	ee'	(E, ee')	(O, ee')
O	eo'	(E, eo')	(O, eo')
	oe'	(E, oe')	(O, oe')
	oo'	(E, oo')	(O, oo')

$u_i = (s_1, s_2, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots)$

NORMAL FORM

1/2	ee'	eo'	oe'	oo'
E	1,1	1,1	2,0	2,0
O	0,2	0,0	0,2	0,0

EXTENSIVE FORM DOESN'T (NECESSARILY) INVOLVE TEMPORAL PRIORITY, BUT IS RATHER A SEQUENCE OF INFORMATION REVELATION.

STRATEGY PROFILES DEFINE UNIQUE PATHS THROUGH THE GAME TREE.

EXTENSIVE FORM REDUCES TO NORMAL FORM (PLAYERS, STRATEGIES, UTILITY FUNCTIONS).

STRATEGY PROFILES:

$$U_1(S_1, S_2) = \begin{cases} 1 \text{ IF } (E, ee') \text{ OR } (E, eo') \\ 0 \text{ IF ALL THE REST } (4) \\ 2 \text{ IF } (E, oe') \text{ OR } (E, oo') \end{cases}$$

EACH EXTENSIVE FORM HAS A UNIQUE NORMAL FORM, BUT EACH NORMAL FORM CAN BE REPRESENTED BY MULTIPLE EXTENSIVE FORMS.

THE SAME NORMAL FORM CAN REPRESENT DIFF. STRATEGIC SITUATIONS

COMMON PRACTICE: START WITH THE NORMAL FORM TO ELICIT INTUITIONS, THEN MOVE TO THE EXTENSIVE (DYNAMIC) MODEL.

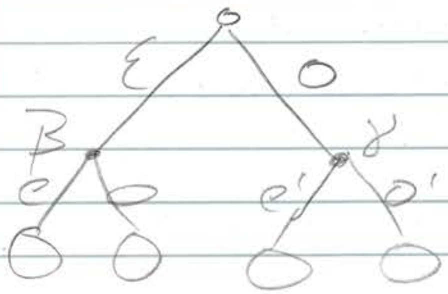
(NORMAL FORM GAMES)

MIXED STRATEGIES: S_1 + S_2 (PREV. PAGE) ARE PURE STRATEGIES. A MIXED STRATEGY IS A PURE STRATEGY OVER A PROBABILITY DISTRIBUTION:
$P(ee') = .2$; $P(e, o') = .8$ (MUST SUM TO 1)
REPRESENTED BY SIGMA: $\sigma_1(ee') = .2$

(

BEHAVIOR STRATEGIES: ASSIGNS A DIFFERENT PROB DISTRO AT INFORMATION SETS.

10/22/18



$P(e,e') = .18$
$P(e,o') = .42$
$P(O,e') = .12$
$P(O,O') = .28$

COULD BE ANY TWO NUMBERS

IDENTIFYING WHICH STRATEGIES ARE RATIONAL

A RATIONAL PERSON CHOOSES THEIR MOST-PREFERRED LOTTERY GIVEN THE STRATEGIC CHOICES OF THE OTHER PLAYERS.

AN EQUILLIBRIUM IS A STRATEGY PROFILE IN WHICH NO PLAYER WANTS TO UNILATERALLY DEVIATE. (SPECIFIED BY STRATEGY PROFILE, NOT PAYOFFS)

NOTE: THE PRISONER'S DILEMMA EQUILIBRIUM IS NOT PARETO EFFICIENT - SEE GENERAL EQUILIBRIUM THEORY

(ME)
* THIS IS AN IMPLICIT CRITIQUE OF CAPITALISM
↳ ∃ SOME SITUATIONS WHERE DISTRIBUTED RATIONALITY FAILS TO REACH PARETO-OPTIMAL EQUILLIBRIA.

SOLUTION CONCEPT - DOMINANT-SPACED REASONING
STRICT DOMINANT STRATEGY EQUILLIBRIUM
↳ WHERE BOTH PLAYERS PLAY A STRICTLY DOMINANT STRATEGY

s_i STRICTLY DOMINATES s_i' IF
IF $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$
FOR ALL $s_{-i} \in S_{-i}$ (NOT QUITE COMPLETE SEE 4.1–4.2)

WEAK DOMINANCE: CHANGE $>$ (ABOVE) TO $\geq$ $\rightarrow$ THEN

UTILITY OF A MIXED STRATEGY:

$$U_1(P, L) = P(2) + 1-P(4) > 3$$

$$U_1(P, R) = P(6) + 1-P(2) > 3$$

SOLVE FOR P

		L	R
(P)	U	2,3	6,2
	M	3,1	3,6
(1-P)	D	4,0	2,8

NO DOMINANT PURE STRATEGIES
BUT THERE IS A DOMINANT MIXED STRATEGY

$4 - 2p > 3;\ p < \frac{1}{2}$

$2 + 4p > 3;\ p > \frac{1}{4}$

0 $\quad \frac{1}{4} \quad \frac{1}{2} \quad$ 1

GAME THEORY 4B - NOTES

1\2	L	C	R
U	2,3	1,5	2,3
M	3,1	3,6	3,5
D	4,0	6,2	5,0

PLAYER 1

$u_1(D,L) > u_1(U,L)$ s_1', s_2

$u_1(D,C) > u_1(U,C)$ s_1', s_2'

$u_1(D,R) > u_1(U,R)$ s_1', s_2''

IF ALL 3 ABOVE ARE TRUE, D STRICTLY DOMINATES U

IF D STRICTLY DOMINATES BOTH U AND M, THEN D IS A DOMINANT STRATEGY

$u_1(D,L) > u_1(M,C)$ s_1', s_2

$u_1(D,C) > u_1(M,C)$ s_1'', s_2'

$u_1(D,R) > u_1(M,R)$ s_1'', s_2''

STRICT DOMINANCE - MIXED STRATEGIES

	1\2	L	C	R
(p)	U	2,3	1,5	5,3
	M	3,1	3,6	3,5
(1-p)	D	4,0	6,2	2,0

$u_1(p,L) > u_1(M,L)$

$u_1(p,C) > u_1(M,C)$

$u_1(p,R) > u_1(M,C)$

$u_1(p,L) = p(2) + (1-p)(4)$	$4-2p > 3$
$u_1(p,C) = p(1) + (1-p)(6)$	$6-5p > 3$
$u_1(p,R) = p(5) + (1-p)(2)$	$2+3p > 3$
* → REVIEW INEQUALITY MANIPULATION	$\frac{1}{3} < p < \frac{1}{2}$

DOMINANT STRATEGY (DOMINANCE) REASONING IS CRUDE AND MINIMAL → IT'S A THRESHOLD CONDITION FOR RATIONALITY

(THIS SHOULD COME AFTER ITERATED DOMINANCE)

(PEIRL - VISUAL CAUSAL INFERENCE)

ITERATED DOMINANCE

1\2	L	C	R
U	4,3	5,1	6,2
M	2,1	8,4	3,6
D	3,0	9,6	2,8

S_2
R DOMINATES C (NOT L)
SO P_2 WILL NOT PLAY C
(SO WE WIPE OUT C)
NOW P_1 HAS A DOMINANT STRATEGY -> PLAY U
(SO WIPE OUT M + D)
AND WE END UP @ (4,3)

6.1 BEST RESPONSE (TO A SPECIFIC STRATEGY)

1\2	L	C	R
U	*1,1	-2,2	4,3*
M	0,3	3,1	5,4*
D	*1,5*	*4,3	*6,2

$u_1(\overset{s_1}{U},\overset{s_2}{L}) \geq u_1(\overset{s_1'}{M},\overset{s_2}{L})$

$u_1(U,L) \geq u_1(D,L)$ s_1'', s_2

U IS A BEST RESPONSE TO L
(SO IS D)

BEST RESPONSE ALGORITHM
-> PLAYER 1, COLUMN WISE (P_2 ROW WISE)

$S = \{D, L\}$ IS A CASE OF MUTUAL BEST RESP.
EVEN THOUGH PLAYER 2 DOESN'T HAVE ANY DOMINANT STRATEGY, C IS NEVER A B.R.
(SO WE ELIMINATE C AND M) (M WAS DOMINATED)

WHAT REMAINS IS THE SET OF RATIONALIZABLE STRATEGIES

10/ /18

GAME THEORY 4B - NOTES (CONT'D)

3-PLAYER STAG HUNT

1	S		H	
	S	H	S	H
2/3 S	$\boxed{*3,*3,*3}$	0,0,3	2,0,0	*2,0,2*
H	0,2,0	0,2,2*	*2,2,0	$\boxed{*1,*1,*1}$

P = 2 1 3

$u_2(S,SS) \geq u_2(H,SS)$

$3 \geq 2$

$u_2(S,HS) \geq u_2(H,HS)$

$0 \not\geq 2$

NASH EQUILIBRIUM

THE SOLELY RATIONAL STRATEGY PROFILE (WHERE EACH PLAYER IS PLAYING A BEST RESPONSE). (ABOVE IN BOXES)

"A STRATEGY PROFILE OF MUTUAL BEST RESPONSES"

BEST RESPONSE CORRESPONDENCE

$$BR_1(S_{-1}) = \begin{cases} S & \text{IF } (S,S) \\ H & \text{IF } (S,H)(H,S) \\ H & \text{IF } (H,H) \end{cases} \quad \text{OR} \quad \begin{cases} S & \text{IF } (SS) \\ H & \text{OTHERWISE} \end{cases}$$

	L	C	R
U	*5,1	1,4*	1,0
M	3,2	*4,0	$\boxed{*3,5*}$
D	4,3	$\boxed{*4,4*}$	0,4*

$$BR_1(S_2) \begin{cases} \{U\} & \text{IF } S_2 = U \\ \{M,D\} & \text{IF } S_2 = M \\ \{M\} & \text{IF } S_3 = D \end{cases}$$

NASH EQUILIBRIA ARE ROBUST TO ITERATIVE REASONING AND DON'T REQUIRE THE COMMON KNOWLEDGE ASSUMPTION (IT IS REQUIRED FOR ITERATED DOMINANCE).

10/29/12

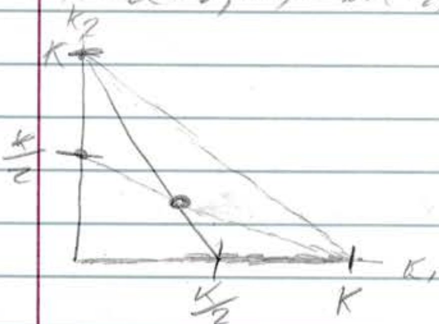
GAME THEORY 5A - NOTES

CONDITIONS FOR WRITING A UTILITY FUNCTION
(RATHER THAN ENUMERATING BEST RESPONSES)

2 FIRMS THAT CONSUME CLEAN AIR

$u_1(K_1, K_2) = \ln(K_1) + \ln(K - K_1 - K_2)$

$u_2(K_2, K_1) = \ln(K_2) + \ln(K - K_2 - K_1)$



BEST RESPONSE FUNCTION? (CORRESP.)
↳ FIRST DERIVATIVE MAXIMIZES

$$\frac{\partial u_1}{\partial K_1} = \frac{1}{K_1} - \frac{1}{K - K_1 - K_2}$$

2 EQUATIONS, 2 UNKNOWNS

$$\begin{cases} K_1^*(K_2) = \dfrac{K - K_2^*}{2} & \text{(BEST RESPONSE CORRESPONDENCE)} \\ K_2^*(K_1) = \dfrac{K - K_1^*}{2} \end{cases}$$

↳ SOLVE FOR $K_1^* + K_2^*$; $K_1^* + K_2^* = K/3$

PROCEDURE :
1) FIND BEST RESPONSE
2) TAKE 1ST DERIVATIVE
3) SOLVE FOR K_1, K_2 (MAXIMIZE)
4) SOLVE SYSTEM (FIND INTERSECTION)

MIXED STRATEGY NASH EQUILIBRIA

1\2	H	T
H	*1, -1	-1, 1*
T	-1, 1*	*1, -1

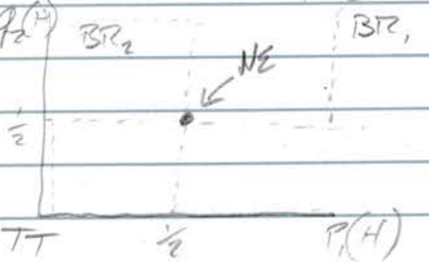
NO PURE STRATEGY NE

P_2: q ON H, $(1-q)$ ON T
GIVEN THIS, FIND A q FOR P_1
SUCH THAT P_1 IS INDIFFERENT BTW H + T

$u_1(H, q) = q - 1 + q = 2q - 1$
$u_1(T, q) = -q + 1 - q = 1 - 2q$
$\Big\}$ $2q - 1 \geq 1 - 2q$, $q \geq 1/2$

$u_2($
$u_2($

MIXED STRATEGY EQUILIBRIUM: $P = \frac{1}{2}, q = \frac{1}{2}$

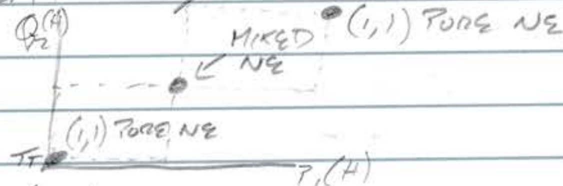


WHEN $P = \frac{1}{2}$, P_2 IS INDIFFERENT BETWEEN ANY MIX OF q + $1-q$ (AND VICE VERSA)

↳ THESE ARE THE VERTICAL + HORIZONTAL LINES

1\2	H	T
H	$1^*, 1^*$	0, 0
T	0, 0	$1^*, 1^*$

→ M.P W/ COORDINATION



$$u_1(\overset{P}{P}, \overset{q}{q}) = \frac{1}{2}\left[\frac{1}{2}(1) + \frac{1}{2}(0)\right] + \frac{1}{2}\left[\frac{1}{2}(0) + \frac{1}{2}(1)\right] = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

IN THIS CASE, THE PURE STRATEGY EQ. ARE PARETO-SUPERIOR TO THE MIXED STRATEGY EQ.

PLAYERS ONLY PLAY A MIXED STRATEGY WHEN THEY

* ARE INDIFFERENT AMONG THEIR PURE STRATEGIES.

BOTH PLAYERS NEED NOT PLAY MIXED STRATEGIES

↳ ONLY ONE CAN, AND IT'S STILL A M.S.S.

ROCK PAPER SCISSORS

1\2	R	P	S
R	0,0	$-1, 1^*$	$^*1, 1$
P	1, -1	0, 0	-1, 1
S	-1, 1	$^*1, -1$	0, 0

- PLAYER 2 RULES OUT R (ASSUME)

THEN P_1 WILL NEVER PLAY P

AS A RESULT, P_2 WILL EXCLUDE R

PLAYERS NEVER PLAY A MIX W/ A DOMINATED STRATEGY

THE UPSHOT OF THIS EXAMPLE IS THAT PLAYERS WILL ALWAYS BE FORCED TO MIX EQUALLY AMONG ALL THE NONDOMINATED STRATEGIES AVAILABLE TO THEM.

↳ AND NON-BEST RESPONSE

SYMMETRIC MIXED STRATEGY EQUILIBRIUM
3-PLAYER MIXED STRATEGIES (SEE HK NOTES)

1	E		N	
2\3	E	N	E	N
E	$(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$	$(0^*, 0^*, 0^*)$	$(0^*, 0^*, 0^*)$	$(0^*, 1^*, 0)$
N	$(0^*, 0^*, 0^*)$	$(1^*, 0^*, 0^*)$	$(0^*, 0, 1)$	$(\frac{2}{3}^*, \frac{2}{3}^*, \frac{2}{3}^*)$

$P_2 = P_3 = P$

$P_1: \Pr(EE) = P^2$

$\Pr(EN) = P(1-P)$

$\Pr(NE) = P(1-P)$

$\boxed{\Pr(NN) = (1-P)^2}$

$u_1(E_1, P_2, P_3) = P^2(-\frac{1}{3}) + P(1-P)(0) + P(1-P)(0) + (1-P)^2 - 1$

$= 1 - 2P + \frac{2}{3}P^2$

$u_1(N, P_2, P_3) = \frac{4}{3}P + \frac{2}{3}P^2$ — SET EQUAL?

$1 - 2P + \frac{2}{3}P^2 = \frac{4}{3}P + \frac{2}{3}P^2$ SEE NOTES

QUADRATIC EQUATION: YIELDS 2 SOLUTIONS
↳ USE THE ONE(S) BTW. 0 + 1
(COULD BE BOTH, 1, OR NEITHER)

10/31/18

HOW TO INTERPRET MIXED-STRATEGY EQUILIBRIA
SEE NOTES ↳ REVELATION CONDITION: IF IT WERE REVEALED THAT THE OTHER PLAYER HAS A PARTICULAR

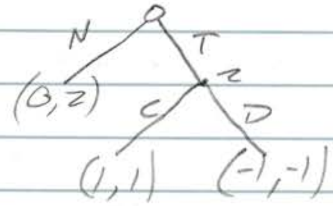
NASH'S FUNDAMENTAL THEOREM - EVERY FINITE GAME HAS AT LEAST ONE MIXED STRATEGY NE.

STRATEGIC RATIONALITY - GOAL-DIRECTED AGENTS CHOOSE A UTILITY-MAXIMIZING STRATEGY GIVEN WHAT OTHER AGENTS ARE DOING.

GAME THEORY 5B - NOTES

10/31/18

DYNAMIC GAMES W/ COMPLETE INFORMATION



1\2	C	D
N	0,2*	$\boxed{*0,2*}$
T	$\boxed{*1,1*}$	-1,-1

SUSTAINED BY A NON-CREDIBLE THREAT

THE NASH EQUILIBRIUM PREDICTS THAT N,D IS A BEST RESPONSE → BUT IF P_2 WERE CHOOSING, THEY WOULD PICK C.

NASH EQUILIBRIUM DELIVERS COUNTERINTUITIVE RESULTS IN DYNAMIC SITUATIONS.

PATH OF PLAY → THE SET OF NODES REACHED WITH POSITIVE PROBABILITY IN A PARTICULAR STRATEGY PROFILE. IF THAT STRATEGY PROFILE IS IN EQUILIBRIUM, WE CALL IT THE EQUILIBRIUM PATH OF PLAY. ABOVE, N,D IS ONLY A NASH EQUILIBRIUM BECAUSE IT <u>IGNORES</u> WHAT'S HAPPENING OFF THE PATH OF PLAY.

THE PATH OF PLAY ENDS WHEN THERE'S A DISCONTINUITY IN THE PATH AS WRITTEN ON A TREE. THIS DISTINGUISHES BETWEEN THE ACTUAL PATH (ROOT TO END) AND THE COUNTERFACTUAL PATH.

3.1 SUBGAME PERFECTION - SEQUENTIAL RATIONALITY
↳ A STRATEGY IS SEQUENTIALLY RATIONAL IF IT IS RATIONAL AT EVERY NODE IN THE PATH OF PLAY.

FOR A PLAYER'S STRATEGY TO BE SEQUENTIALLY RATIONAL, THEY MUST PLAY A BEST RESPONSE AT EACH NODE.

CONTINUATION GAME VS. CONTINUATION PAYOFF → A NODE WHERE A PLAYER IS CHOOSING BETWEEN A CERTAIN PAYOFF AND A FURTHER SUBGAME. (BACKWARDS INDUCTION)

A SEQUENTIALLY RATIONAL STRATEGY IS ONE THAT CONSISTENTLY CHOOSES THE HIGHEST CONTINUATION PAYOFF.
A PROFILE IS SEQUENTIALLY RATIONAL IF IT REQUIRES PLAYERS TO PLAY A BR @ EACH NODE.

REFINEMENT: PLAYERS SHOULD PLAY A NASH EQ. AT EACH CONTINUATION GAME.
SEQUENTIALLY RATIONAL NEs ARE A SUBSET OF THE FULL SET OF NEs.

BACKWARDS INDUCTION - BEGIN AT THE FINAL SET OF NODES, & FIND SEQUENTIALLY RATIONAL RESPONSES AT EACH, THEN TRACE THE PATH OF PLAY.

SEQUENTIALLY RATIONAL STRATEGY PROFILES MUST ACCOUNT FOR COUNTERFACTUALS. BECAUSE WHAT HAPPENS OFF THE PATH OF PLAY DETERMINES THE PATH OF PLAY

FOR EVERY FINITE GAME OF PERFECT INFORMATION THERE EXISTS (AT LEAST ONE) A BACKWARDS INDUCTION SOLUTION IF NO NODES HAVE EQUAL PAYOFFS, THAT SOLUTION IS UNIQUE.

PROPER SUBGAME - STARTS AT A SINGLETON NODE + INCLUDES ALL SUCCESSOR NODES, AND AVOID CUTTING ANY INFORMATION SETS. THE ENTIRE GAME COUNTS AS A PROPER SUBGAME (DISTINCT FROM PROPER SUBSET)

PLAYERS PLAY THEIR BEST RESPONSE IN EACH PROPER SUBGAME.

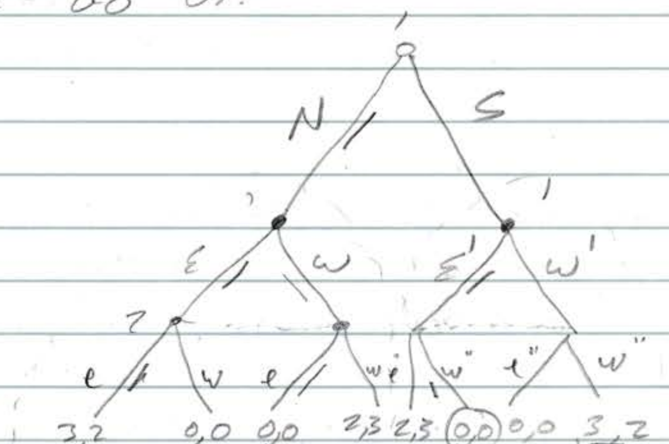
11 / 5 / 18

Game Theory Week 6A

Subgame Perfect Equilibrium
- ↳ Players play an NE in each proper subgame
- ↳ Strategy profile, restricted to each subgame, is a Nash equilibrium.

$\sigma = (N E \underline{W'}, w \underline{e'})$ (Restriction to subgame)

Procedure: Go in reverse, take the lowest subgame, find its Nash Eq., preserve as we go up.



$(N E \bar{E}, \underline{e} \bar{e}')$

$(N E W', e W')$

NE: (must include)

(E, e) (E', e')

(W, w) (W', w')

(Model normal form to find)

	ee'	ew'	we'	ww'
NEE'	*3,2*	*3,2*	0,0	0,0
NEW'	*3,2*	*3,2*	0,0	0,0
NWE'	0,0	0,0	*2,3*	2,3*
NWW'	0,0	0,0	*2,3	2,3*
SEE'	2,3*	0,0	*2,3*	0,0
SEW'	0,0	3,2*	0,0	*3,2
SWE'	2,3*	0,0	*2,3	0,0
SWW'	0,0	*3,2*	0,0	*3,2

Circled equilibria fit the NE requirements above; the others don't

$\boxed{SPE \subseteq NE}$

Definition: Continuation value?

SINGLE DEVIATION PRINCIPLE - NASH EQUILIBRIA ARE SUBGAME-PERFECT IF NEITHER PLAYER HAS AN INCENTIVE TO DEVIATE FROM THE PATH OF PLAY. (ONE DEVIATION IS ENOUGH TO RULE OUT SUBGAME PERFECTION).

↳ IMPORTANT TO HOLD EVERYTHING ELSE FIXED.

SUBGAME PERFECT ONLY IF NO PLAYER HAS A SINGLE PROFITABLE DEVIATION.

USEFUL FOR REPEATED GAMES (EVEN INFINITE GAMES)

MULTI-STAGE GAMES - DYNAMIC INTERACTIONS

↳ PAYOFFS COME AS A STREAM

SELF-CONTAINED INTERACTIONS WITH PAYOFFS UNFOLDING OVER TIME.

L = LEAVE ALONE
P = PUNISH

SEE PROP. 5.1

$t = 1$

1\2	C	D
C	4,4	-1,5
D	*5,-1	*1,1*

$t = 2$ (L = LEAVE ALONE, P = PUNISH)

1\2	L	P
L	*0,0*	-3,-1
P	-1,3	*-2,-2*

PERCEPTIONS @ LATER STAGES INFLUENCE / CONDITION BEHAVIOR @ EARLIER STAGES

PLAYERS WILL MAXIMIZE UTILITY (GIVEN OTHERS' STRATEGIES) OVER ALL STAGES OF THE GAME.

PLAYERS WILL DISCOUNT BENEFITS OVER TIME

$x^1, x^2, x^3 \ldots x^T$; $u(x^T, \delta) =$

$$u(x^T, \delta) = u_1(x^1) + \delta u_1(x^2) + \delta\delta u_1(x^3) + \ldots$$

$$+ \ldots + \delta^{T-1} u_1(x^T)$$

$$(x + \delta x + \delta^2 x + \delta^3 x + \ldots \delta^{T-1} x) = \sum_{t=1}^{T} \delta^{t-1} x (1-\delta)$$

"We're not trying to model actual people, we're modeling rational people."

11/5/18

All of which reduces to:

$$x(1-\delta^T) = \left(\sum_{t-1}^{T} \delta^{T-1} x\right) \qquad \delta \in [0,1]$$

1 - perfect patience

0 - perfect myopia

Reduced further:

Total present value:
(of payoff stream)

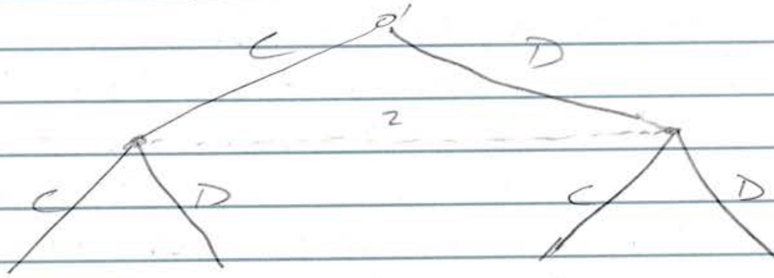
$$\frac{x(1-\delta^T)}{1-\delta} = \sum_{t-1}^{T} \delta^{t-1} x$$

$$\frac{x}{1-\delta}$$

(This is geometric discounting, not hyperbolic $(\beta\delta^T)$)

Total present value of the games on prev. page

$1 + \delta(0 \text{ or } -2)$

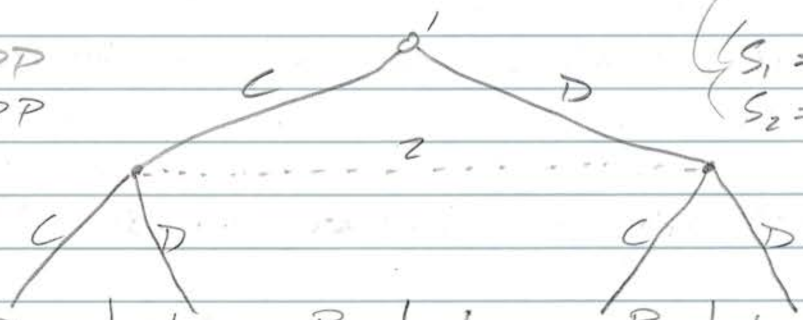


	L	P	
L	$(4+\delta(0), 4+\delta(0))$	$(4+\delta(-3), 4+\delta(-1))$	(Left)
P	$(4+\delta(-1), 4+\delta(-3))$	$(4+\delta(-2), 4+\delta(-2))$	

	L	P	
L	$(1+\delta(0), 1+\delta(0))$	$(1+\delta(-3), 1+\delta(-1))$	(Right)
P	$(1+\delta(-1), 1+\delta(-3))$	$(1+\delta(-2), 1+\delta(-2))$	

For this game, an equilibrium strategy rewards cooperation.

$S_1 = D, PPPP$
$S_2 = D, PPPP$



SUBGAME PERFECT
$S_1 = D, LLLL$
$S_2 = D, LLLL$
(S_1, S_2)

	L	P	L	P	L	P	L	P
L	$4+\partial 0$	$4+\partial(-3)$	$-1+\partial(0)$	$-1+\partial(-3)$	$5+\partial(0)$	$5+\partial(-3)$	$1+\partial(0)$	$1+\partial(-3)$
	$4+\partial 0$	$4+\partial(-1)$	$5+\partial(0)$	$5+\partial(-1)$	$-1+\partial(0)$	$-1+\partial(-1)$	$1+\partial(0)$	$1+\partial(-1)$
P	$4+\partial(-1)$	$4+\partial(-2)$	$-1+\partial(-1)$	$-1+\partial(-2)$	$5+\partial(-1)$	$5+\partial(-2)$	$1+\partial(-1)$	$1+\partial(-2)$
	$4+\partial(-3)$	$4+\partial(-2)$	$5+\partial(-3)$	$5+\partial(-2)$	$1+\partial(-3)$	$-1+\partial(-2)$	$1+\partial(-3)$	$1+\partial(-2)$

FIRST-STAGE PAYOF + DISCOUNTED 2ND STAGE PAYOFF
↳ WHAT PLAYERS DO IN THE FIRST STAGE DOESN'T MATTER FOR THE SUBGAME.

THE REASONING THAT LED TO THE NE IN THE FIRST SUBGAME CAN BE APPLIED TO ALL 4.

WHAT ARE THE SUBGAME-PERFECT EQUILIBRIA THAT MOTIVATE COOPERATION?
$S_1 = \{C, L$ $\qquad S_2 = \{C, L$

A SINGLE PROFITABLE DEVIATION IS ENOUGH TO SHOW ~ SUBGAME PERFECT.

STRATEGY PROFILES MUST NOW INCLUDE ∂ VALUES
"IF ∂ IS $\geq \frac{1}{2}$, THEN..." (BOUNDARY CONDITION)

11/7/18

GAME THEORY 6B NOTES

1. SHOW WHAT NEEDS TO HAPPEN OFF THE PATH TO KEEP BOTH PLAYERS ON THE PATH.

2. COMPARE CONTINUATION VALUES (CETERIS PARIBUS)

PLAYERS WILL ALWAYS PLAY NASH EQUILIBRIA

"KNIFE-EDGE CONDITION" → $\delta = 0$ (NO PATIENCE) OR $\delta = 1$ (MAXIMAL PATIENCE)

MULTIPLE NE_s IN THE FINAL STAGE ARE REQUIRED FOR "HISTORY-CONTINGENT" STRATEGIES - CREDIBLE THREATS REQUIRE MULTIPLE NE_s.

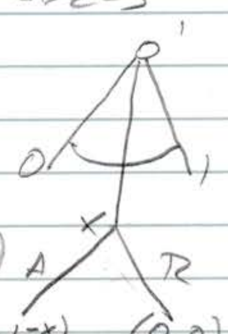
↳ THIS IS WHY WE DIDN'T DO TWO PRISONERS' DILEMMAS.

INFINITELY REPEATED P.D. RESULTS IN
1) MUTUAL COOPERATION; 2) INFINITE SUBGAME PERF. EQ.

DYNAMIC BARGAINING MODELS

ULTIMATUM GAME

PLAYER 1 OFFERS X
PLAYER 2 ACCEPTS (1-X)
OR REJECTS (0,0)



STRATEGY PROFILE
S_1 = (ONE STRATEGY)
S_2 = (INFINITE SUBGAMES)

P_2 IS PLAYING A CUT-POINT STRATEGY (MONOTONICALLY INCREASING IN x)

P_2: BR_1: {PROPOSE x^*} BR_2: {ACCEPT IF $x^* \geq 0$; REJECT OTHERWISE}

$BR_2(x)$ = {ACCEPT IF $x < 1$; ACCEPT OR REJECT IF $x = 1$}

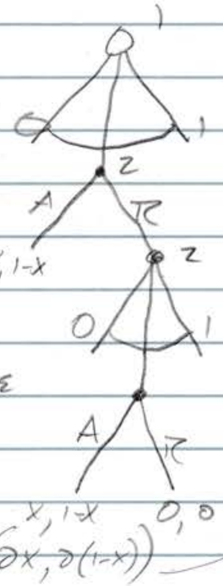
S_2': ACCEPT $x < 1$; REJECT $x = 1$

S_2: ACCEPT ALL X
S_1 = PROPOSE $x = 1$

But in the case of S_2' $\left(\begin{array}{l}\text{Accept } x < 1 \\ \text{Reject } x = 1\end{array}\right)$

B_1 doesn't have a best response
$\Rightarrow$ b/c $1 - \partial > 1 - \varepsilon$ for all epsilon.
So not a subgame perfect Eq.

New Game:
(Additional stage)



(0, δ)

We can simply put (0, ∂) here

Steps
(4) S_1 : At $t = 1$, propose $1 - \delta$
(2) At $t = 2$, accept all x
(3) S_2 : @$t = 1$, accept if $x \leq 1 - \delta$, reject otherwise
(1) At $t = 2$, propose $x = 0$

* See Figure 10 in notes
No exam questions on bargaining / infinite games

11/14/18

Game Theory 7A

Bayesian Games

Candidate Entry Deterrence

TYPES

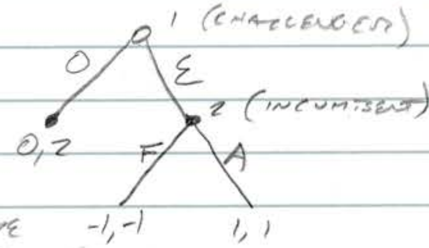
1\2	F	A
O	0,2	0,2
E	-1,-1	1,1

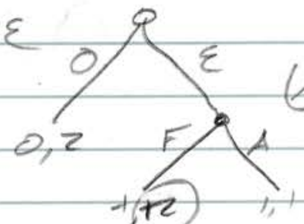
Incumbent has 2 types → Timid →

↓

Aggressive

Subgame Perfect



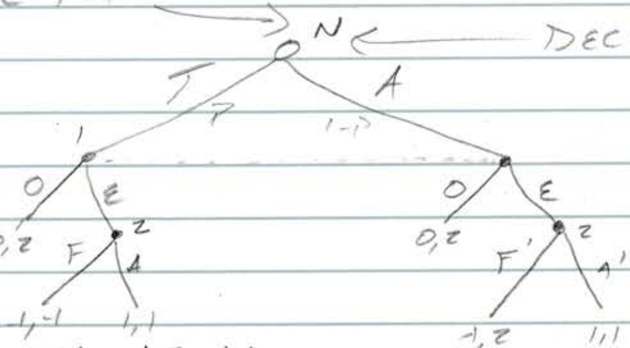


Same except for P_2 payoff in F

Timid: P

Aggressive: 1-P

Decides on type



Type-contingent strategy

	FF	FA	AF	AA
O	0,2	0,2	0,2	0,2
E (Probability-weighted)	(-1, 2-3P)	(1-2P, 1-2P)	(2P-1, 2-P)	(1, 1)

These 3 are Bayesian Nash-Equilibria

Use backwards induction but weight the probabilities.

Associate singleton nodes w/ types

Types represent uncertainty about either (what?)

THETA Θ_1 = Player 1's type space = $\{\Theta_1^1, \ldots, \Theta_1^k\}$

$P_1 = P_{\theta_1}$ = Player 1's beliefs about other players' types = (Θ_{-i})

PLAYERS WILL USE BAYES' RULE TO UPDATE BELIEFS. (PRIOR BELIEFS BECOME POSTERIOR BELIEFS VIA AN UPDATING MECHANISM LIKE BAYES' RULE).

H - HYPOTHESIS (T OR F) $\quad$ E - EVIDENCE (PERTINENT TO HYPOTHESIS)

CONDITIONAL PROBABILITY OF HYPOTHESIS GIVEN EVIDENCE

$$Pr(H|E) = \frac{Pr(H + E)}{Pr(E)} = \frac{Pr(H) \times Pr(E|H)}{Pr(H)\,Pr(E|H) + Pr(\sim H)\,Pr(E|\sim H)}$$

SPECIFY ACTIONS FOR EACH TYPE

$u_1(E, FA; \Theta_2)$ (UTILITY OF PLAYING E AGAINST FA GIVEN PLAYER 2'S TYPE).

$$= P(-1) + (1-P)(1)$$

$\phi_1(\Theta_2 = T) \quad u_1(s_1 = E, s_2 = FA \; ; \Theta_2 = A)$

11/18/18

GAME THEORY 8A

THE PLAYER w/ MULTIPLE TYPES WILL CHANGE PAYOFFS ACROSS TABLE

	L	R
L	2,2	0,0
R	0,0	4,4

	L	R
L	0,2	2,0
R	4,0	0,4

A WELL-FORMED STRATEGY REQUIRES A SPECIFICATION FOR EVERY TYPE

IN THIS EXAMPLE, PLAYER 1 HAS 2 POSSIBLE TYPES (SO THEY HAVE 4 STRATEGIES)

TYPES: COOPERATIVE (P) + CONTRARIAN (1-P)

PLAYER 1

AB	L	R
LL	2P	2(1-P)
LR	4-2P	0
RL	0	2+2P
RR	4-4P	4P

PLAYER 2

	L	R
LL	2	0
LR	2P	4-4P
RL	2-2P	4P
RR	0	4

WHEN THE VALUE OF P ISN'T SPECIFIED, WE CAN STILL FIND SOME BEST RESPONSES (0 < 4)
AND WE CAN IDENTIFY CUT POINTS (P > 2/3)
WE CAN CREATE AN INEQUALITY FOR EACH ROW/COL.
DRAW A NUMBER LINE: LL RL | LL | LL LR

0 1/3 2/3 1

THEN WRITE A BR FUNCTION:

$$BR_2 = \begin{cases} L \text{ IF } LL, RL \\ R \text{ IF } LR, RR \end{cases}$$

IT'S POSSIBLE TO DO THIS ANALYSIS FOR PLAYER 1
(BUT WE HAVE TO TREAT THE 4 OPTIONS
USING PAIRWISE COMPARISONS)

FOR:

$P<1$ LR IS BR TO L

$P=1$ LL, LR " "

$P \in 0,1$ RL IS BR TO R

$P=1$ RL, RR

$P=0$ LL, RL

F = FIRM

5.8 $\Theta_B = \{L, M, H\}$ $S_F = P \in [0, \infty)$ (P = PRICE OF LOBBYIST)

$S_B(\Theta_B) : [0,\infty) \times \{L, M, H\} \rightarrow \{A, R\}$

FOR INSTANCE, $(P=10, \Theta=M) \rightarrow \{R\}$

(MUST SPECIFY FOR ALL P, Θ)

$$u_F(P, A; \Theta_B) = \begin{cases} 14-P & \text{IF } L \\ 24-P & \text{IF } M \\ 34-P & \text{IF } H \end{cases}$$

$(u_F(P, R; \Theta_B) = 0)$

$$U_B(P, A; \Theta_B) = P \qquad U_B(P, R, \Theta_B) = \begin{cases} 10 & \text{IF } L \\ 20 & \text{IF } M \\ 30 & \text{IF } H \end{cases}$$

WHEN IN DOUBT, START FIGURING OUT BEST RESPONSES

$$BR(\Theta_B) = \begin{cases} A \text{ IFF } P \geq 10 + \Theta_B = L \\ A \text{ IFF } P \geq 20 + \Theta_B = M \\ A \text{ IFF } P \geq 30 + \Theta_B = H \end{cases}$$

$EV(\text{ACCEPTED } P) = \frac{1}{3}(14+24+34) = 24$

$EV(\text{RESERVE}) = \frac{1}{3}(10+20+30) = 20$

BUT THE HIGH TYPE WOULDN'T ACCEPT 20

NEW $EV(\text{ACCEPTED } P) = \frac{1}{2}(14+24) + 0(34) = 19$

11 / 18 / 18

GAME THEORY 8B

IF YOU PURSUE THIS UPDATING LOGIC "ALL THE WAY DOWN", THE FINAL BARGAINING RANGE IS 10 - 14.

$$BNE = S_B^*(\Theta_B) = \begin{cases} A \text{ IFF } P \geq P^* \in [10, 14) & \text{IF } L \\ A \text{ IFF } P \geq 20 \quad " & \text{IF } M \\ A \text{ IFF } P \geq 30 & \text{IF } H \end{cases}$$

$S_F^* = P^* \in [10, 14]$

(INFINITE BNEs BTW 10 + 14)

8B INFINITE TYPES CASE: TYPES DISTRIBUTED 0 — 1

$$U_F(P, S_B; \Theta) = \begin{cases} 0 \text{ IF } \{R\} \\ 3\Theta - P \text{ IF } \{A\} \end{cases}$$

$$U_B(P, S_B; \Theta) = \begin{cases} P \text{ IF } A \\ 2\Theta \text{ IF } R \end{cases}$$

$P \geq 2\Theta$ — LOWER BOUND

$\Theta \leq \hat{\Theta}(P) \equiv \frac{1}{2}P$ (THRESHOLD TYPE) — $T = \frac{1}{2}$ (MIDPOINT OF UNIT INTERVAL)

$0 \leq Pr(A \mid P)(3 \times E_V[\Theta] - P)$

$0 \leq Pr(A \mid P)(\frac{3}{2} - P) \to$ UPPER BOUND

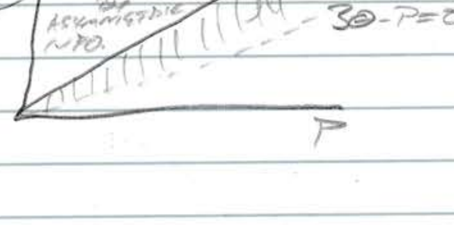
$$\hookrightarrow = \begin{cases} 1 \text{ IF } \Theta \leq \frac{1}{2}P \\ 0 \text{ IF } \Theta > \frac{1}{2}P \end{cases}$$

$3\left(\frac{P}{4}\right) - P = -\frac{1}{4}P$

$BNE = S_F^* = P = 0$

$$S_B^* = \begin{cases} A \text{ IF } \Theta \leq \frac{1}{2}P \\ R \text{ OTHERWISE} \end{cases}$$

(ADVERSE SELECTION)



ASYMMETRIC INFORMATION HAS BIG RAMIFICATIONS FOR WHAT IS RATIONAL.

WRITE TO SG
NEW PENCILS
ARIONE RE: 219
WIENS = SAM
RESPOND TO
TIM + SAT

$$1-(1-4\sqrt{c})^{\frac{1}{2}}$$

PUBLIC GOODS CONTRIBUTION GAME

$e_1 \in \{0,1\}$ $\quad e_2 \in \{0,1\}$ (TWO LEGISLATORS)

$c \in (0,1]$

$$u_i(e_i, e_j, \theta_i) = \begin{cases} \theta_i^2 - c & \text{IF } e_i = 1 \\ \theta_i^2 & \text{IF } e_i = 0 \,\&\, e_j = 1 \\ 0 & \text{IF } e_i = e_j = 0 \end{cases}$$

FIRST, FIGURE OUT BEST RESPONSES:

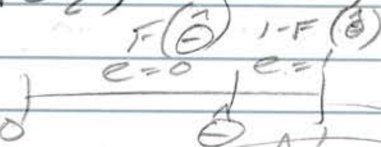
$$\theta_1^2 - c \geq \Pr(e_2 = 1 \mid \theta_2)\,\theta_1^2$$

$$(1 - \Pr(e_2 = 1 \mid \theta_2))\, 0 \text{ — CANCELS}$$

SYMMETRICAL FOR P2

$$\theta_1 \geq \sqrt{\frac{c}{1 - \Pr(e_2 = 1 \mid \theta_2)}} \quad \text{(THRESHOLD TYPE)}$$

$$\hat{\theta}_1 \equiv \sqrt{\frac{c}{1-(1-\hat{\theta})}} = \sqrt{\frac{c}{\hat{\theta}_2}}$$



$$\hat{\theta}_1 \equiv \sqrt{\frac{c}{\hat{\theta}_2}} \qquad \hat{\theta}_2 \equiv \sqrt{\frac{c}{\hat{\theta}_1}}$$

SO: $\hat{\theta}_2^2\,\hat{\theta}_1 = c$

AND: $\hat{\theta}_1 = \hat{\theta}_2$

$$\hat{\theta} = c^{\frac{1}{3}}$$

IF THE PROBLEM IS FULLY SYMMETRIC, THEN $\hat{\theta}_1 = \hat{\theta}_2$

GOAL: EXPRESS θ IN TERMS OF c

$$\hat{\theta}_1(\hat{\theta}_2) = \begin{cases} c^{\frac{1}{3}} & \text{IF } \hat{\theta}_2 \geq c \\ 1 & \text{IF } \hat{\theta}_2 < c \end{cases}$$

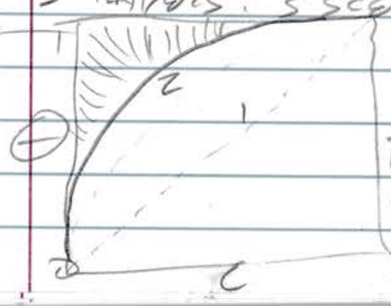
BNE: IF $\hat{\theta}_i \geq c$

$$S_i^*(\theta) = \begin{cases} e_i = 1 & \theta_i \geq \hat{\theta}_i \\ e_i = 0 & \theta_i < \hat{\theta}_i \end{cases}$$

FOR ALL i, THEN

$$S_i^*(\theta) = \begin{cases} e_i = 1 & \text{IF } \theta \geq c^{\frac{1}{3}} \\ e_i = 0 & \text{OTHERWISE} \end{cases}$$

3-PLAYERS: 3 SCENARIOS FOR FREE-RIDING



E.U. IS NEGATIVE ON .... (1): $\theta^2 - c = 0$

BREAK-EVEN TYPE: (2): $\theta^2 = c$

PROBABILITY OF PROVISION: $\quad \hat{\theta} = c^{\frac{1}{3}}$

$$\pi = (1-c^{\frac{1}{3}})^2 + 2(1-c^{\frac{1}{3}})\,c^{\frac{1}{3}} \qquad \hat{\theta}^3 = c$$

$$= 1 - c^{\frac{2}{3}}$$

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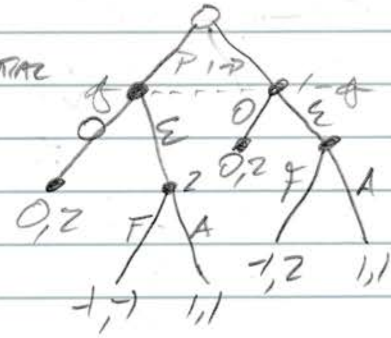
GAME THEORY – 9A

WE USE SEQUENTIAL RATIONALITY TO ELIMINATE $\frac{2}{3}$ OF THE NASH EQUILIBRIA



SEQ. ~~RATIONAL~~ BNE $(P = \frac{2}{3})$

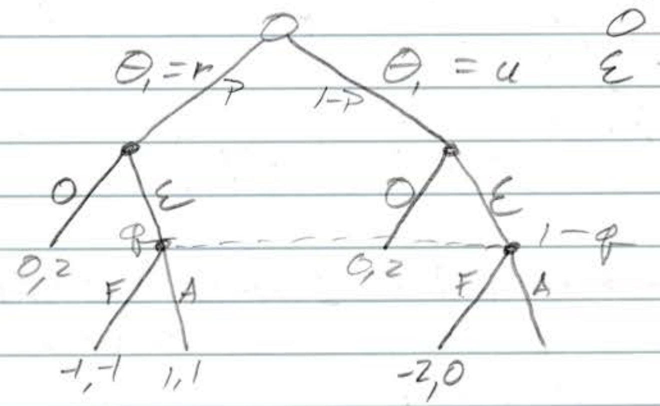
× (O, FF)
× (O, FA)
✓ (E, AF)

BEST RESPONSE (WITH TYPES) MUST TAKE INTO ACCOUNT UNCERTAINTY ABOUT THE OTHER PLAYERS' TYPES $(q, 1-q)$.

NOW P_1 HAS THE TYPES: $P=\frac{1}{2}$ FF FA AF AA

O 0,2 0,2 0,2 0,2

E



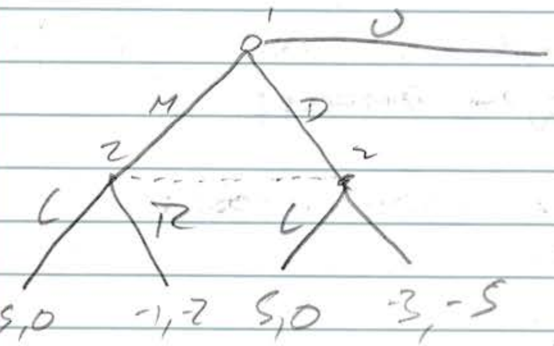
OOF IS SUBGAME PERFECT BUT NOT SEQ. RATIONAL

$P=\frac{1}{2}$	F	A
OO	$\boxed{0,2}$	$0,2$
OE	$-1,1$	$\frac{1}{2},\frac{3}{2}$
EO	$-\frac{1}{2},\frac{1}{2}$	$\frac{1}{2},\frac{3}{2}$
EE	$-\frac{3}{2},-\frac{1}{2}$	$\boxed{\frac{1}{2},1}$

2 BNE: (OO, F) + (EE, A)

THERE'S NO BELIEF FOR WHICH FIGHT IS A BEST RESPONSE.

NEW CONCEPT OF RATIONALITY: PLAYERS ARE PLAYING BEST RESPONSES AT EACH OF THEIR INFORMATION SETS GIVEN THEIR BELIEFS RE: OTHER PLAYERS' STRATEGIES.



	L	R
U	0,5	$\boxed{0,5}$
M	$\boxed{5,0}$	-1,-2
D	$\boxed{5,0}$	-3,-5

THESE EQUILIBRIA MUST BE SUBGAME PERFECT B/C ONLY ONE PROPER SUBGAME.

"SYSTEM OF BELIEFS" -> A WELL-FORMED PROBABILITY DISTRIBUTION ASSIGNED TO EACH INFORMATION SET.

$$q = Pr(\theta_1 = r \mid \sigma_1(\varepsilon \mid r))$$
$$1-q = Pr(\theta_1 = r \mid \sigma_1(\varepsilon \mid r))$$
$$\downarrow$$
$$q = Pr(\theta_1 = r \mid \varepsilon)$$
$$1-q = Pr(\theta_1 = \ ?\ )$$

$$q = Pr(\theta = r \mid \varepsilon) = \frac{Pr(\theta = r)\, Pr(\varepsilon \mid r)}{Pr(\varepsilon)}$$
$$= \frac{\frac{1}{2} \times 1}{\frac{1}{2}} = 1$$

LEARNING ONLY TAKES PLACE (BELIEFS ONLY CHANGE FROM THEIR INITIAL VALUES) WHEN

FINAL FORMAT
FEDERALIST
?

11 / 26 / 18

PERFECT BAYESIAN EQUILIBRIUM (σ^*, μ)
APPLIES TO STRATEGY PROFILES

CHARACTERIZING A PBE INVOLVES STATING
THE STRATEGY REQUIREMENTS AND STATING
WHAT THE BELIEFS ARE.

PROB. DISTRO
ASSIGNED TO EACH
INFORMATION SETS.

REQUIREMENTS:
#1 - μ IS A WELL-DEFINED SYSTEM OF BELIEFS
#2 - μ IS CONSISTENT W/ BAYES' RULE AT
ALL INFORMATION SETS ON THE PATH OF σ^*.
#3 - μ IS CONSISTENT WITH BAYES' RULE AT ALL
OFF-PATH INFORMATION SETS WHERE POSSIBLE
#4 - σ_i^* MUST BE SEQUENTIALLY ~~RATIONAL~~ SOMETIMES UNDEFINED
RATIONAL AT EACH INFORMATION SET
(ON OR OFF THE PATH) GIVEN σ_{-i}^* + μ_i.

(σ^*, μ) IS A PBE IFF #s 1-4 OBTAIN.

WEAK PBE ELIMINATES #3 - FOR INFORMATION
SETS OFF THE PATH, ANYTHING GOES.

(PREV. PAGE)
EXAMPLE: $u_1(F, q) \geq u_1(A, q)$ (NEVER TRUE)

BACKWARDS INDUCTION

$BR_2(q) = \{A\}$ FOR ALL VALUES OF q

$BR_1(s_2; \theta_1) \begin{cases} \{E\} \text{ IF } \theta_1 = r \\ \{E\} \text{ IF } \theta_1 = w \end{cases}$
$= \{E\}$ FOR ALL θ_1

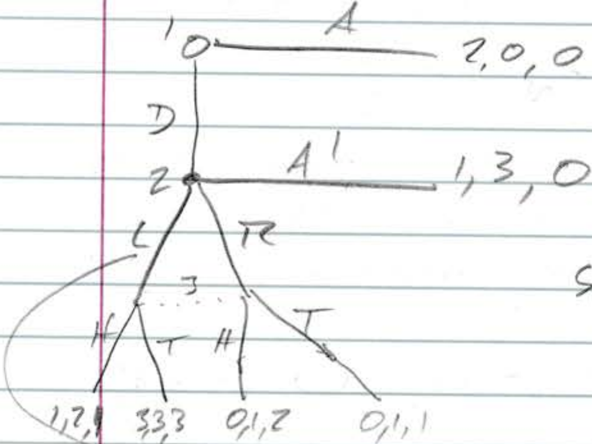
$\sigma^* \begin{cases} s_1(\theta_1) = E \text{ FOR ALL } \theta_1 \\ s_2 = A \end{cases}$

Now we need beliefs:

Since $S_1(\theta_1)$, $\mu = q = p$ (no updating took place: types are equally likely).

So $(\theta\theta, F)$ can be eliminated (not PBE) b/c it doesn't conform to σ^*

Requirement #3 is tricky - see Ex. 4.0.2 in the notes.



$s_1 = A$	H	T
	$\boxed{2,0,0}$	$\boxed{2,0,0}$
A'	$\boxed{2,0,0}$	$\boxed{2,0,0}$
L	$\boxed{2,0,0}$	$\boxed{2,0,0}$
R		

$s_1 = D$	H	T
A'	1,3,0	1,3,0
L	1,2,1	$\boxed{3,3,3}$
R	0,1,2	0,1,1

L strictly dominates R, so player 2 will never play R. Player 3 should know this.

The example above is wrong - he re-did it on 1/26.

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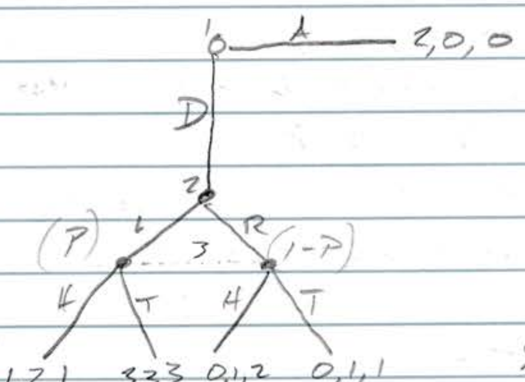
Game Theory 9B



(B)NE: (A,L,H)
(A,R,H)
(A,R,T)
(D,L,T)

Subgame Perfect: (D,L,T)
(in each subgame)

Off-the-path strategies, once updated by Bayes' Rule, constrain possible equilibria.

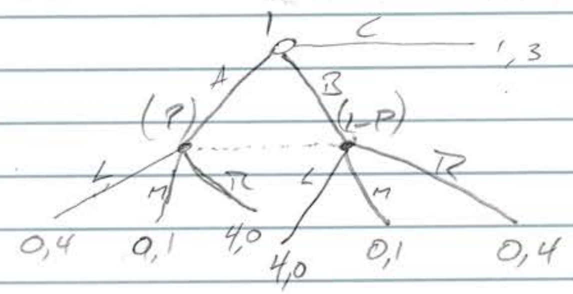
PBE: (D,L,T) + (P=1)
(must have both parts)

$$BR_3(p) = \begin{cases} \{H\} & \text{if } p < \frac{1}{3} \\ \{H,T\} & \text{if } p = \frac{1}{3} \\ \{T\} & \text{if } p > \frac{1}{3} \end{cases}$$

Possibilities for P may be constrained by off-the-path strategies

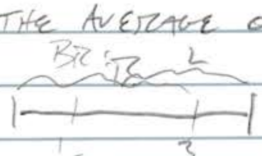
* Procedure: Find the SPEs, then ask if there are any beliefs that justify those strategies.

* [See 4.0.2] - Start w/ BRs as a function of beliefs



	L	M	R
A	0,4	0,1	4,0
B	4,0	0,1	0,4
C	1,3	$\boxed{1,3}$	1,3

If we treat P as a lottery between L + R, then the average of that mix will be better than M.



$p(4) \geq 1$ $4 - 4p \geq 1$ ⇒ This rules out the NE at C,M

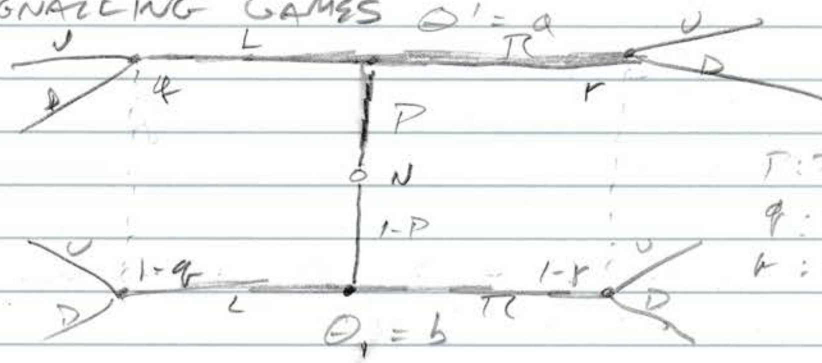
$p \geq 1/4$ $3 \geq 4p$

$\frac{3}{4} \geq p$

SO WE TRY A MIXED STRATEGY: $r = \frac{1}{2}$; $q = \frac{1}{2}$
(BECAUSE THE TOP-LEFT 4 CELLS = MATCHING PENNIES)
↳ RESULTS IN $EU_{1+2} = 2$

SO WE HAVE A NEW PBE: $\sigma_1 = r = \frac{1}{2}$; $\sigma_2 = q = \frac{1}{2}$ $p = 1/2$

SIGNALLING GAMES



T: TYPES
q: BELIEFS
r: BELIEFS

PLAYERS SEE THE OTHER PLAYERS' ACTIONS, BUT THEY DON'T KNOW WHICH TYPE PERFORMED IT.

SEPARATING STRATEGY - TYPES PERFORM DIFFERENT ACTIONS.
IF THIS RESULTS IN AN EQUILIBRIUM, IT IS A SEPARATING EQUILIBRIUM OR A POOLING EQ

PERFECTLY INFORMATIVE / UNINFORMATIVE
POOLING EQUILIBRIUM -

SEPARATING EQUILIBRIUM -

IMPERFECTLY INFORMATIVE
SEMI - SEPARATING EQUILIBRIUM -

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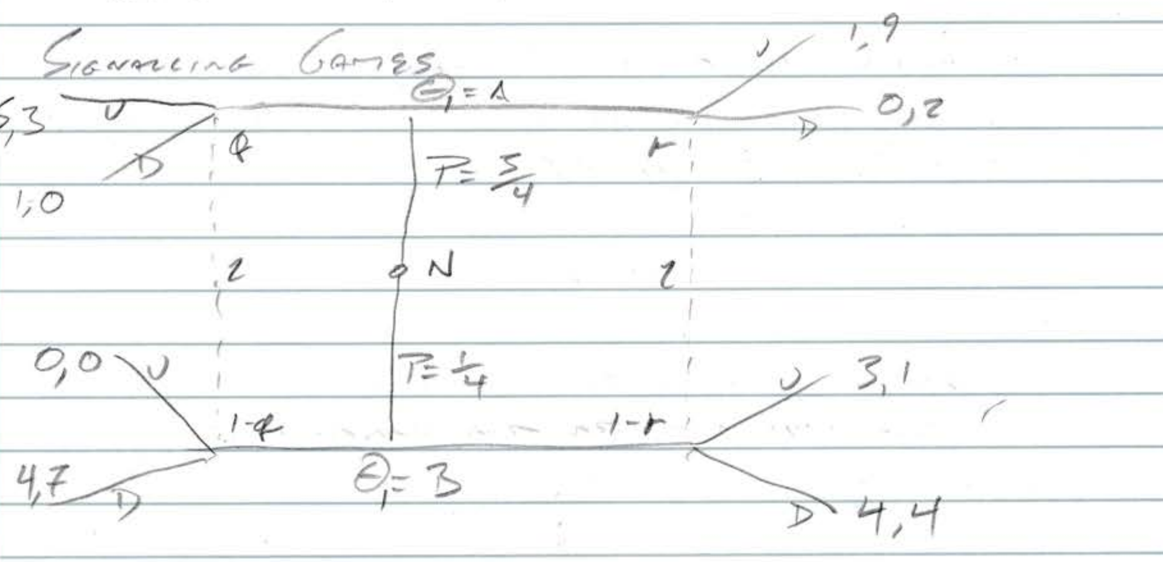
Game Theory 10A

Signalling Games



Solve for Player 2's Best Response as a function of their information set.

#1 $U_2(U, q) \geq U_2(d, q)$

$3q + 0(1-q) \geq 0q + 7(1-q)$

$3q \geq 7 - 7q$

$10q \geq 7 \;;\; q \geq \frac{7}{10}$

So $BR_1 =$

$BR_2(q) = \begin{cases} \{U\} & q > \frac{7}{10} \\ \{U, d\} & q = \frac{7}{10} \\ \{d\} & q < \frac{7}{10} \end{cases}$

#2 $U_2(U, r) \geq U_2(d, r)$

$9r + (1-r) \geq 2r + 4(1-r)$

$8r + 1 \geq 4 - 2r$

$10r \geq 3 \;;\; r \geq \frac{3}{10}$

$BR_2(r) = \begin{cases} \{U\} & r > \frac{3}{10} \\ \{U, d\} & r = \frac{3}{10} \\ \{d\} & r < \frac{3}{10} \end{cases}$

Look for Pooling Equilibria - both types take the same action. Two in this case: (give no information on types)

$q = Pr(A|L) = \frac{Pr(A)Pr(L|A)}{Pr(L)} = q = \frac{3}{4}, \; r = \text{UNDEF.}$

$(\sigma_1(A) = L, B = L)$

So: Against $\{L\}$, P_2 plays $\{U\}$

Now we look for the strategy that Player 2 would have to play <u>off the path</u> to sustain the pooling equilibrium.

So LL is not a pooling equilibrium

NOW WE CHECK FOR A SECOND POOLING EQUILIBRIUM AT R, R ($\theta = A$ PLAYS R
$r = 3/4$, q = UNDEF. $\theta = B$ PLAYS R)
WE CAN RULE OUT u ($5 > 1$) AND d ($3 > 1$)
SO RR IS NOT A POOLING EQUILIBRIUM.

NOW WE CHECK FOR SEPARATING EQUILIBRIA.
THERE WILL ALSO BE TWO HERE: L, R AND R, L
FIRST WE CHECK L, R.
SINCE WE KNOW θ_A WILL PLAY L, THEN $q = 1$
$r = 0$

SO AGAINST L, $\sigma_1 = u$; AGAINST R, $\sigma_1 = d$
WHICH MEANS THIS IS SSE AS LONG AS THERE ARE NO PROFITABLE DEVIATIONS. (THERE IS ONE?)

NOW WE CHECK R, L : $q = 0$, $r = 1$
SO AGAINST L, $\sigma_1 = d$; AGAINST R, $\sigma_1 = u$
NEITHER PLAYER

SEMI-SEPARATING: $q = Pr(A|L) = \dfrac{Pr(A)Pr(L|A)}{Pr(L)}$
$(Pr\, L = Pr(A)Pr(L|A) + Pr(B)Pr(L|B))$

SOLVE THIS, AND YOU GET AN ANSWER WITH S IN IT
(S IS THE MIXED STRATEGY PLAYED BY ONE OF P1'S TYPES)
↳ THEN SET THIS EQUAL TO THE q - VALUE (3/4)

q IS A FUNCTION OF THE PROBABILITY OF PLAYING L.

Ask TE: Grading
Re: Final Coverage

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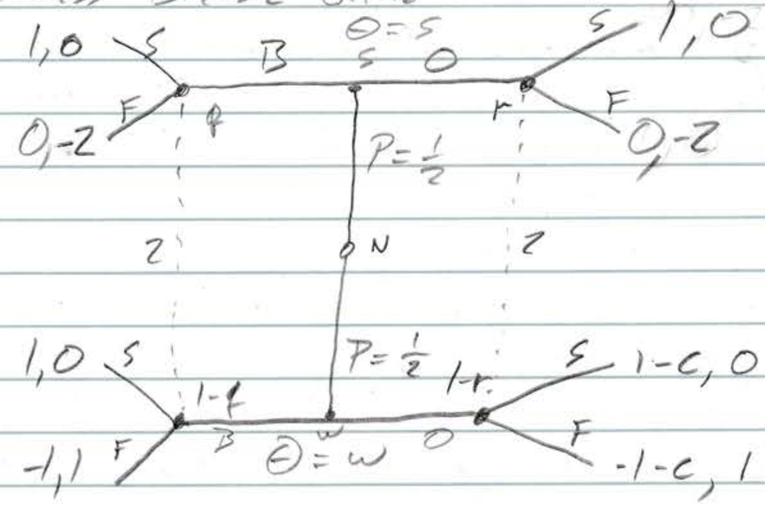
Game Theory 10B

"Princess Bride Game"

S
$0 \geq -2q+1-q$
$q \geq \frac{1}{3}$



F
$-2r \geq 1-r$
$r \geq 1$
(Never best response)

$BR_2(q) = \{F\}$ for $\forall r$; $BR_2(r) = \begin{cases} \{F\} & \text{if } q < \frac{1}{3} \\ \{F,S\} & \text{if } q = \frac{1}{3} \\ \{S\} & \text{if } q > \frac{1}{3} \end{cases}$

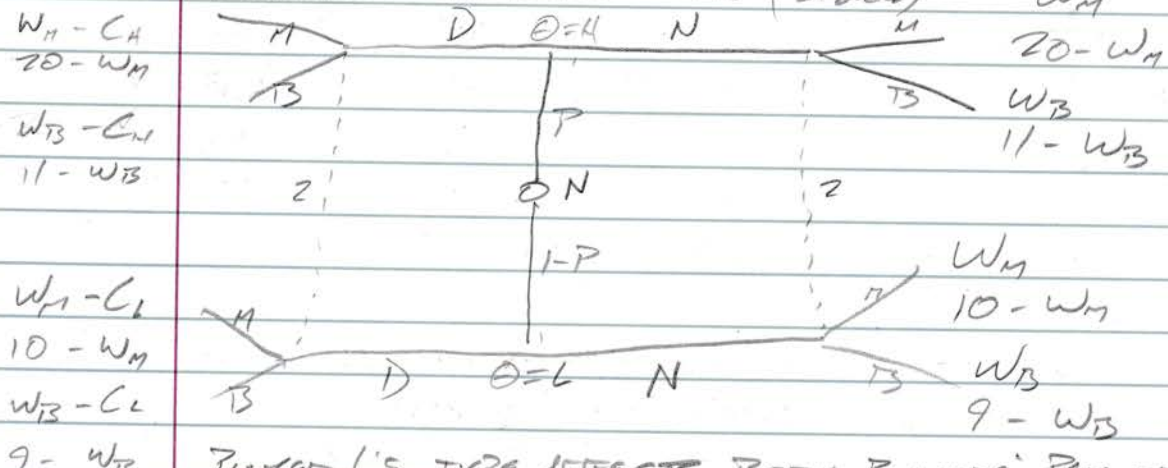
Against O, $s_2 = S$; Against B, $s_2 = F$

$\left.\begin{array}{l} 1-c \geq -1 \\ 2 \geq c \end{array}\right\}$ Type w has no incentive to deviate as long as $c \leq 2$. This sustains the eq.

c values <u>must</u> be specified as part of a PBE

"If c is sufficiently high, they separate. If c is sufficiently low, they pool."

EDUCATION SIGNALLING GAME (SPENCE)



PLAYER 1'S TYPE AFFECTS BOTH PLAYERS' PAYOFFS.

P_1 = EMPLOYEE, P_2 = MANAGER

ASSUMPTIONS: $C_L > C_H \geq 0$; $W_M > W_B$

SPECIFY THE EQUILIBRIUM WE WANT TO SOLVE FOR BEFORE WORKING OUT THE BEST-RESPONSE FUNCTION (OTHERWISE WE'LL HAVE TWO UNKNOWN PARAMETERS)

SO WE CONSIDER A STRATEGY FOR PLAYER 1:

$\sigma_1(H) = D$ SO Q + R ARE: $q = 1$; $r = 0$

$\sigma_1(L) = N$ $20 - W_M \geq 11 - W_B$; $9 \geq W_M - W_B$

$$BR_2(q=1) = \begin{cases} \{M\} \text{ IF } 9 \geq W_M - W_B & \text{- ASSUME FOR NOW} \\ \{B\} \text{ IF } 9 < W_M - W_B \end{cases}$$

$10 - W_M \leq 9 - W_M$; $1 \leq W_M - W_B$ | $9 \geq W_M - W_B \geq 1$

$W_M - C_H \geq W_B$; $W_M - W_B \geq C_H$ (SUSTAINS SEPARATION)

SO: P_2 : $9 \geq W_M - W_B \geq 1$

P_1 : $C_L \geq W_M - W_B \geq C_H$

GRAPH THE PARAMETER SPACE

$$BR_2 = \begin{cases} \text{AGAINST D, PLAY M} \\ \text{AGAINST N, PLAY B} \end{cases}$$

IF $9 \geq C_L \geq W_M - W_B \geq C_H \geq 1$, THEN THIS EQ. IS BNE